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MATHEMATICS **9709/13**

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MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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Question	Answer	Marks	Guidance
1	$\frac{1}{2}n\left[-24+(n-1)6\right] \sim 3000$ Note: $\sim$ denotes <u>any</u> inequality or equality	**M1**	Use correct formula with RHS ≈ 3000 (e.g. 3010).
	$(3)\left(n^2-5n-1000\right)\ (\sim 0)$	**A1**	Rearrange into a 3-term quadratic.
	$n \sim 34.2\ (\&-29.2)$	**A1**	
	35. Allow $n \geqslant 35$	**A1**	
		4	
2	$ax+3a=-\frac{2}{x} \rightarrow ax^2+3ax+2\ (=0)$	***M1**	Rearrange into a 3-term quadratic.
	Apply $b^2-4ac>0$ SOI	**DM1**	Allow $\geqslant$. If no inequalities seen, **M1** is implied by 2 correct final answers in a or x.
	$a<0,\ a>\frac{8}{9}$ (or 0.889) OE	**A1 A1**	For final answers accept $0>a>\frac{8}{9}$ but not $\leqslant$, $\geqslant$.
		4	

Question	Answer	Marks	Guidance
3(i)	$6C3\left(\frac{2}{x}\right)^3(-3x)^3$ SOI also allowed if seen in an expansion	**M1**	Both x's can be missing.
	-4320 Identified as answer	**A1**	Cannot be earned retrospectively in **(ii)**.
		2	
3(ii)	$6C2\left(\frac{2}{x}\right)^4[(-)3x]^2$ SOI clearly identified as critical term	**M1**	Both x's and minus sign can be missing.
	$15a\times16\times9-their4320(=0)$	**A1 FT**	FT on *their* 4320.
	$a=2$	**A1**	
		3	

Question	Answer	Marks	Guidance
4	$f'(x)=\left[\left(\frac{3}{2}\right)(2x-1)^{1/2}\right]\times[2]-[6]$	**B2, 1, 0**	Deduct 1 mark for each [...] incorrect.
	$f'(x)<0$ or $\leqslant 0$ or $=0$ SOI	**M1**	
	$(2x-1)^{1/2}<2$ or $\leqslant 2$ or $=2$ OE	**A1**	Allow with k used instead of x
	Largest value of k is $\frac{5}{2}$	**A1**	Allow $k\leqslant\frac{5}{2}$ or $k=\frac{5}{2}$ Answer must be in terms of k (not x)
		5	

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Question	Answer	Marks	Guidance
5(i)	$\cos\theta + 4 + 5\sin^2\theta + 5\sin\theta - 5\sin\theta - 5\ (=0)$	**M1**	Multiply throughout by $\sin\theta + 1$. Accept if $5\sin\theta - 5\sin\theta$ is not seen
	$5\left(1-\cos^2\theta\right) + \cos\theta - 1\ (=0)$	**M1**	Use $s^2 = 1 - c^2$
	$5\cos^2\theta - \cos\theta - 4 = 0$ AG	**A1**	Rearrange to AG
		3	
5(ii)	$\cos\theta = 1$ and -0.8	**B1**	Both required
	$\theta = [0°, 360°],\quad [143.1°],\quad [216.9°]$	**B1 B1 B1 FT**	Both solutions required for 1st mark. For 3rd mark FT for $(360° - \textit{their}\,143.1°)$ Extra solution(s) in range (e.g. 180º) among 4 correct solutions scores $\frac{3}{4}$
		4	

Question	Answer	Marks	Guidance
6(i)	$y = \frac{2}{x^2 - 1} \Rightarrow x^2 = \frac{2}{y} + 1$ OE	**M1**	
	$x = (\pm)\sqrt{\frac{2}{y} + 1}$ OE	**A1**	With or without x/y interchanged.
	$\mathrm{f}^{-1}(x) = -\sqrt{\frac{2}{x} + 1}$ OE	**A1**	Minus sign obligatory. Must be a function of x.
		3	

Question	Answer	Marks	Guidance
6(ii)	$\left(\frac{2}{x^2-1}\right)^2+1=5$	**B1**	
	$\frac{2}{x^2-1}=(\pm)2$ OE OR $x^4-2x^2=0$ OE $x^2-1=(\pm)1 \Rightarrow x^2=2\ (\text{or}\ 0)$ $x=-\sqrt{2}$ or -1.41 only	**B1**	Condone $x^2=0$ as an additional solution
		4	

Question	Answer	Marks	Guidance
7(i)	$\sin^{-1}\left(\frac{3}{5}\right)=0.6435$ AG	**M1**	OR $(PBC=)\cos^{-1}\left(\frac{3}{5}\right)=0.9273 \Rightarrow (ABP=)\frac{\pi}{2}-0.9273=0.6435$ Or other valid method. Check working and diagram for evidence of incorrect method
7(ii)	Use (once) of sector area $=½r^2\theta$	**M1**	
	Area sector $BAP=½\times5^2\times0.6435$ = 8.04	**A1**	
	Area sector $DAQ=½\times½\pi\times3^2$ = 7.07 , Allow $\frac{9\pi}{4}$	**A1**	
		3	

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Question	Answer	Marks	Guidance
7(iii)	*EITHER:* Region = sect + sect – (rect – Δ) or sect – [rect – (sect + Δ)]	**(M1**	Use of correct strategy
	(Area Δ *BPC* =) ½ × 3 × 4 = 6 Seen	**A1**	
	8.04 + 7.07 – (15 – 6) = 6.11	**A1)**	
	OR1: Region = sector *ADQ* – (trap *ABPD* – sector *ABP*).	**(M1**	Use of correct strategy
	(Area trap *ABPD* =) ½ (5 + 1) × 3 = 9 Seen	**A1**	
	7.07 – (9 – 8.04) = 7.07 – 0.96 = 6.11	**A1)**	
	OR2: Area segment *AP*= 2.5686 Area segment *AQ* = 0.5438 Region = segment *AP* + segment *AQ* + Δ*APQ*.	**(M1**	Use of correct strategy
	(Area Δ*APQ* =) ½ × 2 × 3 = 3 Seen	**A1**	
	2.57 + 0.54 + 3 = 6.11	**A1)**	
		3	

Question	Answer	Marks	Guidance
8(i)	*EITHER:* $4 - 3\sqrt{x} = 3 - 2x \rightarrow 2x - 3\sqrt{x} + 1\ (=0)$ or e.g. $2k^2 - 3k + 1\ (=0)$	**(M1**	Form 3-term quad & attempt to solve for $\sqrt{x}$.
	$\sqrt{x} = ½, 1$	**A1**	Or $k = ½$ or 1 (where $k = \sqrt{x}$).
	$x = ¼, 1$	**A1)**	
	OR1: $(3\sqrt{x})^2 = (1 + 2x)^2$	**(M1**	
	$4x^2 - 5x + 1\ (=0)$	**A1**	
	$x = ¼, 1$	**A1)**	
	OR2: $\frac{3-y}{2} = \left(\frac{4-y}{3}\right)^2 \left(\rightarrow 2y^2 - 7y + 5(=0)\right)$	**(M1**	Eliminate x
	$y = \frac{5}{2}, 1$	**A1**	
	$x = ¼, 1$	**A1)**	
		3	

Question	Answer	Marks	Guidance
8(ii)	*EITHER:* Area under line = $\int(3-2x)\,dx = 3x - x^2$	**(B1**	
	$= \left[(3-1) - \left(\frac{3}{4} - \frac{1}{16}\right)\right]$	**M1**	Apply *their* limits (e.g. ¼ → 1) after integn.
	Area under curve $= \int\left(4 - 3x^{1/2}\right)dx \;=\; 4x - 2x^{3/2}$	**B1**	
	$\left[(4-2) - (1-¼)\right]$	**M1**	Apply *their* limits (e.g. ¼ → 1) after integration.
	Required area = $\frac{21}{16} - \frac{5}{4} = \frac{1}{16}$ (or 0.0625)	**A1)**	
	OR: $+/-\int(3-2x) - \left(4 - 3x^{\frac{1}{2}}\right) = +/-\int(-1-2x+3x^{\frac{1}{2}})$	**(*M1**	Subtract functions and then attempt integration
	$+/-\left[-x - x^2 + \frac{3x^{3/2}}{3/2}\right]$	**A2, 1, 0 FT**	FT on *their* subtraction. Deduct 1 mark for each term incorrect
	$+/-\left[-1-1+2-\left(-\frac{1}{4}+\frac{1}{16}+\frac{1}{8}\right)\right] = \frac{1}{16}$ (or 0.0625)	**DM1 A1)**	Apply *their* limits ¼ → 1
		5	

Question	Answer	Marks	Guidance
9(i)	$\overrightarrow{AB} = +/-\begin{pmatrix}-18\\9\\-18\end{pmatrix}$, $\overrightarrow{BC} = +/-\begin{pmatrix}12\\-6\\12\end{pmatrix}$,	**B1 B1**	Allow **i**, **j**, **k** form throughout.
	$\left\lvert\overrightarrow{AB}\right\rvert = 27$, $\left\lvert\overrightarrow{BC}\right\rvert = 18$	**B1 FT** **B1 FT**	FT on *their* $\overrightarrow{AB}$, *their* $\overrightarrow{OD}$.
	$\left\lvert\overrightarrow{CD}\right\rvert = \left(\frac{18}{27}\right)\times 18$ OR $\left(\frac{18}{27}\right)^2 \times 27 = 12$	**B1**	
		5	
9(ii)	$\overrightarrow{CD} = (\pm)\,their\,\frac{18}{27}\times their\ \overrightarrow{BC}$ SOI	**M1**	Expect $(\pm)\begin{pmatrix}8\\-4\\8\end{pmatrix}$.
	$\overrightarrow{OD} = \begin{pmatrix}2\\-3\\-1\end{pmatrix}(\pm)\ their\ \frac{18}{27}\begin{pmatrix}12\\-6\\12\end{pmatrix} = \begin{pmatrix}10\\-7\\7\end{pmatrix}, \begin{pmatrix}-6\\1\\-9\end{pmatrix}$	**M1 A1 A1**	Other methods possible for $\overrightarrow{OD}$, e.g. $\overrightarrow{OB} + \frac{5}{2}\overrightarrow{CD}$, $\overrightarrow{OB} + \frac{1}{2}\overrightarrow{CD}$ (One soln **M2A1**, 2nd soln **A1**) OR $\overrightarrow{OB} + \frac{5}{3}\overrightarrow{BC}$, $\overrightarrow{OB} + \frac{1}{3}\overrightarrow{BC}$ (One soln **M2A1**, 2nd soln **A1**)
		4	

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Question	Answer	Marks	Guidance
10(i)	$ax^2 + bx = 0 \rightarrow x(ax+b) = 0 \rightarrow x = \frac{-b}{a}$	**B1**	
	Find $f''(x)$ and attempt sub *their* $\frac{-b}{a}$ into *their* $f''(x)$	**M1**	
	When $x = \frac{-b}{a}$, $f''(x) = 2a\left(\frac{-b}{a}\right) + b = -b$ MAX	**A1**	
		3	
10(ii)	Sub $f'(-2) = 0$	**M1**	
	Sub $f'(1) = 9$	**M1**	
	$a = 3 \quad b = 6$	***A1**	Solve simultaneously to give both results.
	$f'(x) = 3x^2 + 6x \rightarrow f(x) = x^3 + 3x^2 \ (+c)$	***M1**	Sub *their a, b* into $f'(x)$ and integrate 'correctly'. Allow $\frac{ax^3}{3} + \frac{bx^2}{2}(+c)$
	$-3 = -8 + 12 + c$	**DM1**	Sub $x = -2,\ y = -3$. Dependent on *c* present. Dependent also on *a*, *b* substituted.
	$f(x) = x^3 + 3x^2 - 7$	**A1**	
		6	

Question	Answer	Marks	Guidance
11(i)	Gradient of $AB = \frac{1}{2}$	**B1**	
	Equation of AB is $y = \frac{1}{2}x - \frac{1}{2}$	**B1**	
		2	
11(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x} = ½(x-1)^{-\frac{1}{2}}$	**B1**	
	$½(x-1)^{-\frac{1}{2}} = ½$. Equate *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$ to *their* ½	***M1**	
	$x = 2,\ \ y = 1$	**A1**	
	$y - 1 = ½(x - 2)$ (thro' *their*(2,1) & *their* ½) $\rightarrow\ y = ½x$	**DM1 A1**	
		5	

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Question	Answer	Marks	Guidance
11(iii)	*EITHER:* $\sin\theta = \frac{d}{1} \rightarrow d = \sin\theta$	**(M1**	Where θ is angle between AB and the x-axis
	gradient of $AB = ½ \Rightarrow \tan\theta = ½ \Rightarrow \theta = 26.5(7)°$	**B1**	
	$d = \sin 26.5(7)° = 0.45$ (or $\frac{1}{\sqrt{5}}$)	**A1)**	
	OR1: Perpendicular through O has equation $y = -2x$	**(M1**	
	Intersection with AB: $-2x = ½x - ½ \rightarrow \left(\frac{1}{5}, \frac{-2}{5}\right)$	**A1**	
	$d = \sqrt{\left(\frac{1}{5}\right)^2 + \left(\frac{2}{5}\right)^2} = 0.45$ (or $\frac{1}{\sqrt{5}}$)	**A1)**	
	OR2: Perpendicular through (2, 1) has equation $y = -2x + 5$	**(M1**	
	Intersection with AB: $-2x + 5 = ½x - ½ \rightarrow \left(\frac{11}{5}, \frac{3}{5}\right)$	**A1**	
	$d = \sqrt{\left(\frac{1}{5}\right)^2 + \left(\frac{2}{5}\right)^2}$ = 0.45 (or 1/√5)	**A1)**	

Question	Answer	Marks	Guidance
11(iii)	*OR3:* ΔOAC has area $\frac{1}{4}$ [where $C = (0, -\frac{1}{2})$]	**(B1**	
	$\frac{1}{2} \times \frac{\sqrt{5}}{2} \times d = \frac{1}{4} \rightarrow d = \frac{1}{\sqrt{5}}$	**M1 A1)**	
		3	