Capacitance (Chapter 18):

- Every capacitor has two leads, each connected to a metal plate, where in between there is an insulating material called the **dielectric**; to store energy, these two plates must be given equal and opposite electric charges.

- Uses of capacitors in electrical circuits, other than for the smoothing of direct current:
  - Storing energy
  - Blocking d.c.
  - In oscillator circuits
  - In tuning circuits
  - In timing circuits

- Explain why the capacitor stores energy but not charge:
  - Charges on plates are equal and opposite, hence there is no resultant charge; energy is stored because there is a charge separation.

- The **capacitance** of a capacitor is the charge stored on one plate per unit of potential difference between the plates. Given by the equation:

\[
C = \frac{Q}{V}
\]

  - \( Q \) is the magnitude of the charge on each of the capacitor’s plates
  - \( V \) is the potential difference across it

- **Farad**: The unit of capacitance (abbreviated F). \( 1 \text{ F} = 1 \text{ C} \text{ V}^{-1} \)

- During charging of a capacitor, power supply is used to push electrons from one plate to another; power supply does work on the electrons, increasing their potential energy, which is recovered during discharging.

- The area under a graph of p.d. against charge is equal to work done.

![Graph](image_url)

**Figure 24.7** The area under a graph of voltage against charge gives a quantity of energy. The area in a shows the energy stored in a capacitor; the area in b shows the energy required to drive a charge through a resistor.
Hence for a capacitor, energy stored is given by:

\[ W = \frac{1}{2} Q V \]

\[ W = \frac{1}{2} CV^2 \]

\[ W = \frac{1}{2} \frac{Q^2}{C} \]

- Capacitors in parallel (same p.d. across each capacitor; total charges equal to the sum of charges):
  - The total charge is given by:

\[ Q = Q_1 + Q_2 = C_1 V + C_2 V \]

  - Since \( V \) is the common factor:

\[ Q = (C_1 + C_2)V \]

  - Comparing this with \( Q = C_{total}V \):

\[ C_{total} = C_1 + C_2 + C_3 + \ldots \]

- Capacitors in series (p.d. is divided among capacitors; each capacitor stores same charges):
  - \( V = V_1 + V_2 \)
  - \[ \frac{Q}{C_{total}} = \frac{Q}{C_1} + \frac{Q}{C_2} \]
  - \[ \frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} \]

<table>
<thead>
<tr>
<th>Capacitors</th>
<th>Resistors</th>
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<tbody>
<tr>
<td><img src="image" alt="Capacitors in series" /></td>
<td><img src="image" alt="Resistors in series" /></td>
</tr>
<tr>
<td>store same charge</td>
<td>have same current</td>
</tr>
<tr>
<td>[ \frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots ]</td>
<td>[ R_{total} = R_1 + R_2 + R_3 + \ldots ]</td>
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<td>have same p.d.</td>
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</tr>
<tr>
<td>[ C_{total} = C_1 + C_2 + C_3 + \ldots ]</td>
<td>[ \frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots ]</td>
</tr>
</tbody>
</table>

Table 24.3 Capacitors and resistors compared.
- **Magnetic fields**: A force field in which a magnet, a wire carrying current, or a moving charge experiences a force.
- Magnetic fields are produced by current-carrying conductors (moving charges (e.g. free electrons)) or by permanent magnets (movement of the electrons within the atoms).
- An electromagnet makes use of the magnetic field created by an electric current; which a coil is used to concentrate the magnetic field (field lines are closer together) – also called **solenoid** – creating north pole and south pole; strength can be increased by addition of ferrous core, due to its easily magnetised property.

**Figure 26.2** Magnetic field patterns: a for a bar magnet; b for two attracting bar magnets; c for two repelling bar magnets.

**Figure 26.3** Magnetic field patterns for a solenoid, and b a flat circular coil.

- The **right-hand grip rule** gives the direction of magnetic field lines in an electromagnet.
A current-carrying wire is surrounded by a magnetic field, which will interact with an external magnetic field, giving rise to a force on the conductor, similar to the fields of two interacting magnets:

- The magnetic force created by the **motor effect** (e.g. a current in a coil’s magnetic field interacts with a second magnetic field produced by a permanent magnet):

![Diagram of motor effect](image)

**Figure 26.8** The copper rod is free to roll along the two horizontal aluminium ‘rails’.

**Figure 26.10** In the field of a permanent magnet, a current-carrying conductor experiences a force in accordance with Fleming’s left-hand rule. The fields due to the permanent magnet and the current (left and centre) combine as shown on the right.
The strength of a magnetic field is known as its **magnetic flux density**, $B$, representing the number of magnetic field lines passing through a region per unit area.

Gravitational field strength $g$ at a point is defined as the force per unit mass:

$$g = \frac{F}{m}$$

Electric field strength $E$ is defined as the force per unit positive charge:

$$E = \frac{F}{Q}$$

The **magnetic flux density** at a point in space is the force experienced per unit length by a long straight conductor carrying unit current and placed at **right angles** to the field at that point:

$$B = \frac{F}{IL}$$

**Tesla**: the magnetic flux density is 1T when a wire carrying a current of 1A placed at **right angles** to the magnetic field experiences a force of 1 N per metre of its length.

The force on the conductor is given by:

$$F = BIL$$

Measuring $B$ with a **current balance**:

- The magnetic field is roughly uniform
- The length $L$ of the current-carrying wire measured using ruler
- When there is no current in the wire, the magnet arrangement is placed on the top pan and the balance is zeroed
- When the current, $I$, flows in the wire, ammeter shows the value
The wire will experience an upwards force and, according to Newton’s third law of motion, there is an equal and opposite force on the magnets, hence pushed downwards causing readings on the balance.

\[ F, I \text{ and } L \] are now known, the magnetic flux density \( B \) between the magnets is given by:

\[ B = \frac{F}{IL} \]

- Currents crossing fields:

\[ F = (B \sin \theta)IL \]

or simply:

\[ F = BIL \sin \theta \]

- Figure 26.22a, their magnetic fields circle around, and in the space between the wires there is an extra-strong field, creating repulsive forces on the two wires.
- Figure 26.22b, in the space between the two wires, the magnetic fields cancel out, hence the wires are pushed together; the two forces are equal and opposite to one another (Newton’s third law of motion).

<table>
<thead>
<tr>
<th>Base units</th>
<th>Derived units</th>
<th>because</th>
</tr>
</thead>
<tbody>
<tr>
<td>m, kg, s</td>
<td>newton N = kg m s(^{-2})</td>
<td>( F = ma )</td>
</tr>
<tr>
<td></td>
<td>joule J = kg m(^2) s(^{-2})</td>
<td>( W = Fd )</td>
</tr>
<tr>
<td></td>
<td>watt W = kg m(^2) s(^{-3})</td>
<td>( P = \frac{W}{t} )</td>
</tr>
<tr>
<td>m, kg, s, A</td>
<td>coulomb C = A s</td>
<td>( Q = It )</td>
</tr>
<tr>
<td></td>
<td>volt V = kg m(^2) A(^{-1}) s(^{-3})</td>
<td>( V = \frac{W}{Q} )</td>
</tr>
<tr>
<td></td>
<td>tesla T = kg A(^{-1}) s(^{-2})</td>
<td>( B = \frac{F}{IL} )</td>
</tr>
</tbody>
</table>

Table 26.1 How derived units relate to base units in the SI system.

The force between two 1 kg masses 1 m apart = \( 6.7 \times 10^{-11} \) N

The force between two charges of 1 C placed 1 m apart = \( 9.0 \times 10^9 \) N

The force per meter on two wires carrying a current of 1 A placed 1 m apart = \( 2.0 \times 10^{-7} \) N
Electric force is strongest and gravity is the weakest, however over larger distances and with objects of large mass, the gravitational field becomes the most significant.

Charged particles (Chapter 27 TB):

- The direction of conventional electric current is the direction of flow of positive charge; when electrons are moving, the conventional current is regarded as flowing in the opposite direction.

![Diagram of electron beam and magnetic field](image)

**Figure 27.3** A beam of electrons is deflected as it crosses a magnetic field. The magnetic field into the plane of the paper is represented by the cross in the circle.

- The size of force on a moving charge in a uniform magnetic field depends on:
  - The magnetic flux density $B$ (strength of the magnetic field)
  - The charge $Q$ on the particle
  - The speed $v$ of the particle
  - Hence if motion at right angles to the magnetic field:

  $$F = BQv$$

- Hence if motion at an angle of $\theta$ to the magnetic field:

  $$F = BQv \sin \theta$$

![Diagram of charged particle in magnetic field](image)

**Figure 27.6** The path of a charged particle is curved in a magnetic field.
The two equations \( F = BIL \) and \( F = BQv \) are consistent with one another:

Since current \( I \) is the rate of flow of charge, we can write:

\[
I = \frac{Q}{t}
\]

Substituting in \( F = BIL \) gives:

\[
F = \frac{BQL}{t}
\]

Now, \( \frac{L}{t} \) is the speed \( v \) of the moving particle, so we can write:

\[
F = BQv
\]

For an electron, with a charge of \(-e\), the magnitude of the force on it is:

\[
F = Bev \quad (e = 1.60 \times 10^{-19} \text{ C})
\]

The force on a moving charge is sometimes called ‘the Bev force’; it is this force acting on all the electrons in a wire which gives rise to ‘the BIL force’

When a charged particle moves at right angles to a uniform magnetic field, magnetic force \( F \) is always perpendicular to its velocity, hence \( F \) acts as a centripetal force (force directed towards the centre of the circle):

\[
\text{centripetal force} = \frac{mv^2}{r}
\]

The centripetal force is provided by the magnetic force Bev.

Therefore:

\[
Bev = \frac{mv^2}{r}
\]

Cancelling and rearranging to find \( r \) gives:

\[
r = \frac{mv}{Be}
\]

The equation rewritten in terms of momentum \( p \) of the particle:

\[
p = Ber
\]

The equation \( r = \frac{mv}{Be} \) shows that:

- Faster moving particles move in bigger circles \((r \propto v)\)
- Particles with bigger masses move in bigger circles, due to more inertia \((r \propto m)\)
- Stronger field makes particles move in tighter circles \((r \propto 1/B)\)

The charge-to-mass ratio of an electron involves finding the charge-to-mass ratio \( \frac{e}{m_e} \) known as the specific charge on the electron

Using the equation for an electron travelling in a circle in a magnetic field:

\[
\frac{e}{m_e} = \frac{v}{Br}
\]
B and r are measurable, but v is not; hence cathode-anode voltage ($V_{ca}$) is used, which p.d. causes each electron to accelerate; if an individual electron has charge – $e$ then an amount of work $e \times V_{ca}$ is done one each electron, which is its K.E. as it leaves the anode:

$$eV_{ca} = \frac{1}{2}m_e v^2$$

Where $m_e$ is the electron mass; $v$ is the speed of the electron

Eliminating $v$ from the two equations gives:

$$\frac{e}{m_e} = \frac{2V_{ca}}{r^2B^2}$$

- **Velocity selector:**

![Velocity selector diagram]

*Figure 27.11 A velocity selector – only particles with the correct combination of charge, mass and velocity will emerge through the slit S.*

- **Hall effect:** the production of voltage across a conductor when a current flows through the conductor at right angles to a magnetic field

  > A small current flows through the probe across the ends; when magnetic field is applied, electrons are pushed sideways by the magnetic force, hence accumulating along one side of the probe – **Hall effect** – and lack of electrons on the other side, resulting to an electric field between the two sides

  > The charge is detected by a small voltage across the probe: **Hall voltage**

  > The greater the magnetic flux density, the greater the Hall voltage

![Hall effect diagram]

*Figure 27.12 Electrons are deflected as they move through the Hall probe.*
The electric field strength $E$ is related to the Hall voltage $V_H$ by:

$$E = \frac{V_H}{d}$$

- $d$ is the width of the slice

![Diagram of a semiconductor slice with Hall voltage and electric and magnetic forces on an electron.](image)

**Figure 27.13** a The Hall voltage is measured across the slice of semiconductor. b The forces on an electron when the electric and magnetic forces on it are balanced.

Equating electric and magnetic forces:

$$eE = Bev$$

Substituting for $E$ we have:

$$\frac{eV_H}{d} = Bev$$

- By substituting $v$ from the equation $I = nAe$, where $A$ is the cross-sectional area of the conductor and $n$ is the number density of the conducting particles:

$$\frac{eV_H}{d} = \frac{Bel}{nAe}$$

Making $V_H$ the subject of the equation (and cancelling $e$)
gives:

$$V_H = \frac{Bld}{nAe}$$

But the area of the side face of the conductor $A = d \times t$, where $t$ is the thickness of the slice. Substituting and cancelling gives:

$$V_H = \frac{Bl}{nte}$$

- Where a more general equation can be given by:

$$V_H = \frac{Bl}{nte}$$
• Velocity selection on charged particles:
  ➢ Where an electric force is given by:

\[ F_E = qE \]

  ➢ If the velocity if the particles before entry into the field is \( v \), then they will follow a parabolic path:

![Figure 22.21](https://www.cienotes.com/)

  ➢ During equilibrium between electric force and magnetic force, the particle will pass through the fields not deflected, where forces are given by:

\[ Bqv = qE \quad \text{and} \quad v = \frac{E}{B} \]

![Figure 22.22 Velocity selector](https://www.cienotes.com/)

It is required to select charged ions which have a speed of \( 4.2 \times 10^6 \) m s\(^{-1} \). The electric field strength in the velocity selector is \( 3.2 \times 10^4 \) V m\(^{-1} \). Calculate the magnetic flux density required.

\[ v = \frac{E}{B} \]

\[ B = \frac{3.2 \times 10^4}{4.2 \times 10^6} \]

\[ = 7.6 \times 10^{-3} \text{T} \]

  ➢ This is due to the increase in magnetic force \( Bqv \) when there is an increase in speed, hence they will be deflected