UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
International General Certificate of Secondary Education

CANDIDATE NAME	

CENTRE NUMBER						CANDIDATE NUMBER				

MATHEMATICS **0580/04**

Paper 4 (Extended) **October/November 2009**

2 hours 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator Geometrical instruments
Mathematical tables (optional) Tracing paper (optional)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown below that question.
Electronic calculators should be used.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π use either your calculator value or 3.142.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 130.

For Examiner's Use

This document consists of **21** printed pages and **3** blank pages.

UNIVERSITY *of* CAMBRIDGE
International Examinations

[Turn over

1   Chris goes to a shop to buy meat, vegetables and fruit.

   (a) (i) The costs of the meat, vegetables and fruit are in the ratio

   meat : vegetables : fruit = 2 : 2 : 3.

   The cost of the meat is $2.40.

   Calculate the **total** cost of the meat, vegetables and fruit.

   Answer(a)(i) $ .... [2]

   (ii) Chris pays with a $20 note.

   What percentage of the $20 has he spent?

   Answer(a)(ii) .... % [2]

   (b) The masses of the meat, vegetables and fruit are in the ratio

   meat : vegetables : fruit = 1 : 8 : 3.

   The total mass is 9 kg.

   Calculate the mass of the vegetables.

   Answer(b) .... kg [2]

(c) Calculate the cost per kilogram of the fruit.

Answer(c) $ .... [3]
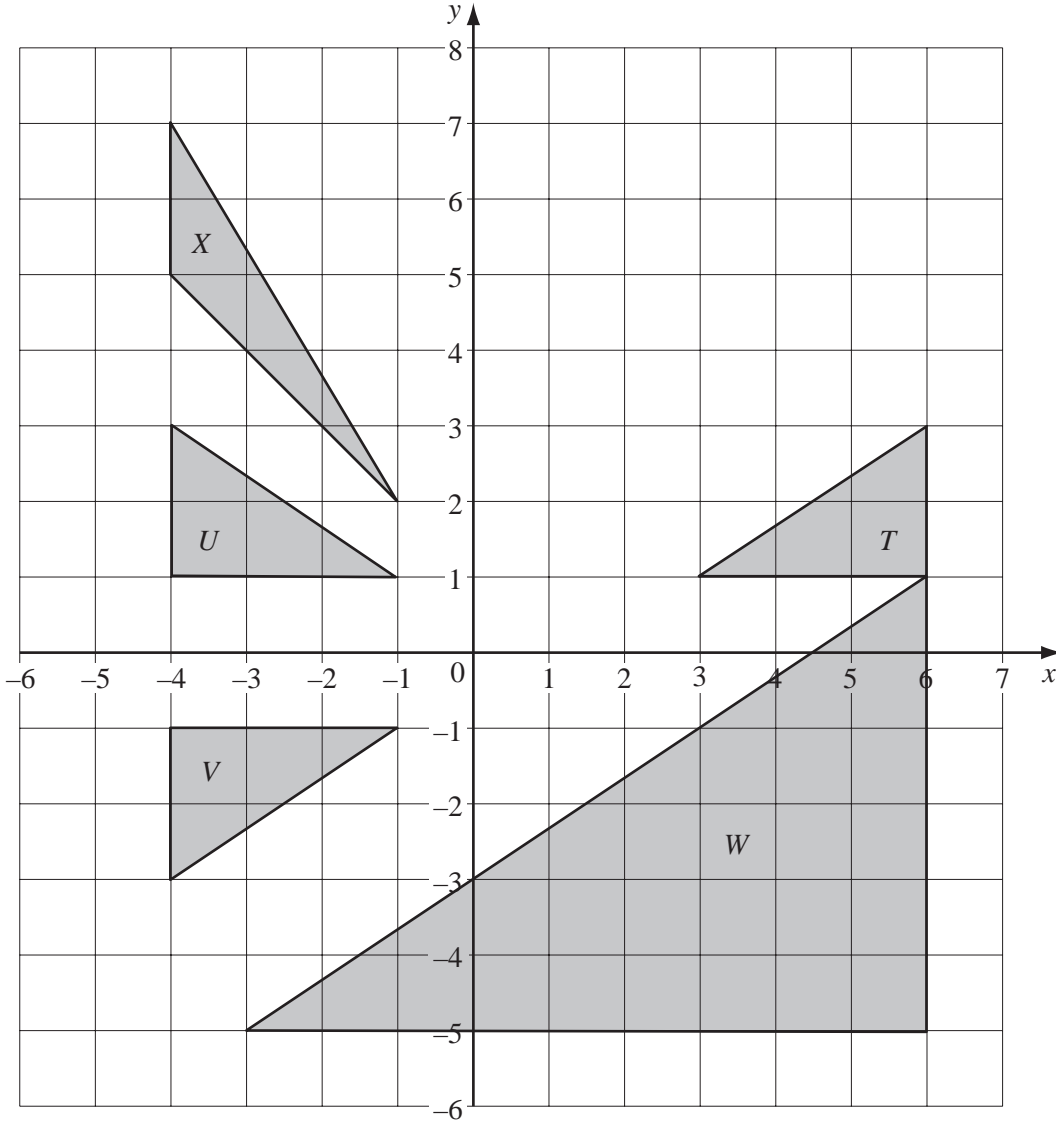
(d) The cost of the meat, $2.40, is an increase of 25% on the cost the previous week.

Calculate the cost of the meat the previous week.

Answer(d) $ .... [2]

2

(a) Describe fully the **single** transformation which maps

   (i) triangle *T* onto triangle *U*,

Answer(a)(i) .... [2]

   (ii) triangle *T* onto triangle *V*,

Answer(a)(ii) .... [3]

(iii) triangle *T* onto triangle *W*,

Answer(a)(iii) .... [3]

(iv) triangle *U* onto triangle *X*.

Answer(a)(iv) .... [3]

(b) Find the matrix representing the transformation which maps

(i) triangle *U* onto triangle *V*,

Answer(b)(i) $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

(ii) triangle *U* onto triangle *X*.

Answer(b)(ii) $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

3
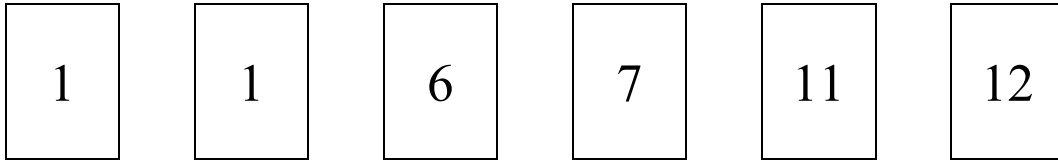
| 1 | 1 | 6 | 7 | 11 | 12 |

Six cards are numbered 1, 1, 6, 7, 11 and 12.

In this question, give all probabilities as fractions.

(a) One of the six cards is chosen at random.

(i) Which number has a probability of being chosen of $\dfrac{1}{3}$?

Answer(a)(i) .... [1]

(ii) What is the probability of choosing a card with a number which is smaller than **at least three of the other numbers**?

Answer(a)(ii) .... [1]

(b) Two of the six cards are chosen at random, without replacement.

Find the probability that

(i) they are both numbered 1,

Answer(b)(i) .... [2]

(ii) the total of the two numbers is 18,

Answer(b)(ii) .... [3]

(iii)   the first number **is not** a 1 and the second number **is** a 1.

Answer(b)(iii) .... [2]

(c) Cards are chosen, without replacement, until a card numbered 1 is chosen.

Find the probability that this happens before the third card is chosen.

Answer(c) .... [2]

(d) A seventh card is added to the six cards shown in the diagram.
The mean value of the seven numbers on the cards is 6.
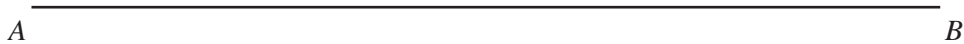
Find the number on the seventh card.

Answer(d) .... [2]

4

C

140 m   180 m

NOT TO
SCALE
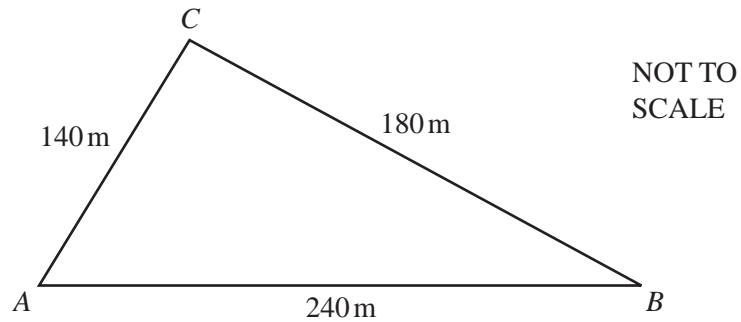
A   240 m   B

The boundary of a park is in the shape of a triangle *ABC*.
AB = 240 m, *BC* = 180 m and *CA* = 140 m.

In part (a), show clearly all your construction arcs.

(a) (i) Using a scale of 1 centimetre to represent 20 metres, **construct** an **accurate** scale drawing of triangle *ABC*. The line *AB* has already been drawn for you.

A   *B*

[2]

(ii) Using a straight edge and compasses only, **construct** the bisector of angle *ACB*.

Label the point *D*, where this bisector meets *AB*. [2]

(iii) Using a straight edge and compasses only, construct the locus of points, inside the triangle, which are equidistant from *A* and from *D*. [2]

(iv) Flowers are planted in the park so that they are nearer to *AC* than to *BC* **and** nearer to *D* than to *A*.

Shade the region inside your triangle which shows where the flowers are planted. [1]

In part (b), use trigonometry.
You must show your working and must NOT use any measurements from your construction in part (a).

(b) (i) Show clearly that angle *ACB* is 96.4°.

Answer(b)(i)

[3]

(ii) Calculate the area of the park.

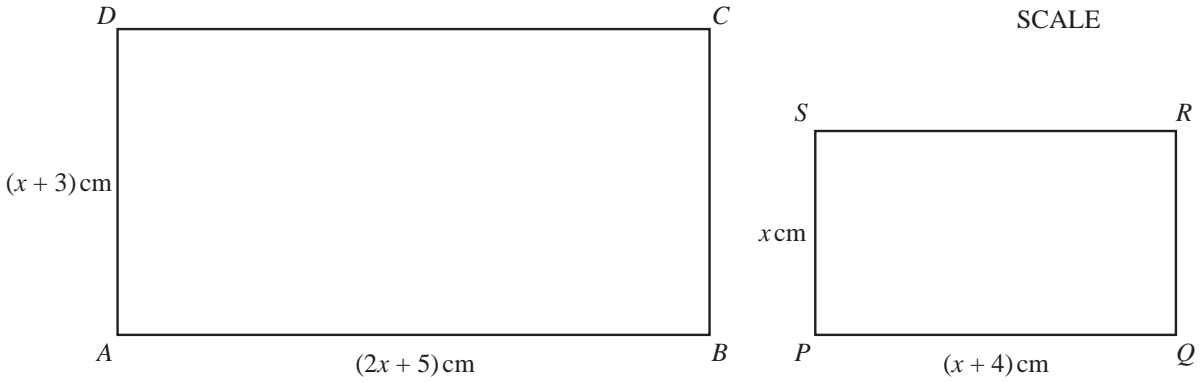*Answer(b)*(ii) .... m^2 [2]

(iii) Use the sine rule to calculate angle *ABC*.

Answer(b)(iii) Angle *ABC* = ... [3]

5

NOT TO SCALE

The diagram shows two rectangles *ABCD* and *PQRS*.

$AB = (2x + 5)$ cm, $AD = (x + 3)$ cm, $PQ = (x + 4)$ cm and $PS = x$ cm.

(a) For one value of x, the area of rectangle *ABCD* is 59 cm^2 more than the area of rectangle *PQRS*.

(i) Show that $x^2 + 7x - 44 = 0$.

Answer(a)(i)

[3]

(ii) Factorise $x^2 + 7x - 44$.

Answer(a)(ii) .... [2]

(iii) Solve the equation $x^2 + 7x - 44 = 0$.

Answer(a)(iii) $x =$ .... or $x =$ .... [1]

(iv) Calculate the size of angle *DBA*.

Answer(a)(iv) Angle *DBA* = ... [2]

(b) For a **different** value of x, the rectangles *ABCD* and *PQRS* are similar.

(i) Show that this value of x satisfies the equation $x^2 - 2x - 12 = 0$.
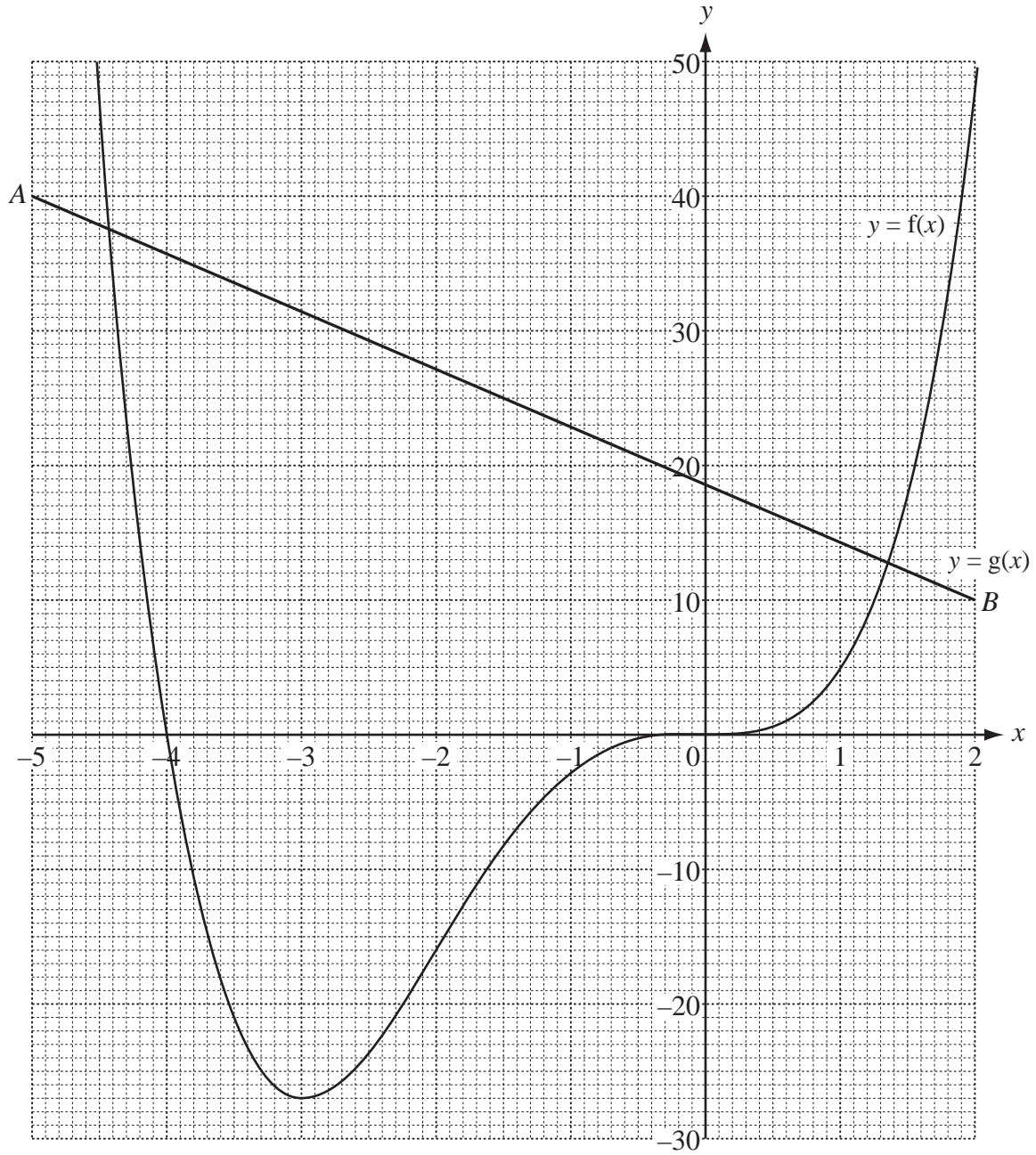
Answer(b)(i)

[3]

(ii) Solve the equation $x^2 - 2x - 12 = 0$, giving your answers correct to 2 decimal places.

Answer(b)(ii) $x =$ .... or $x =$ .... [4]

(iii) Calculate the perimeter of the rectangle *PQRS*.

Answer(b)(iii) .... cm [1]

6

The graphs of $y = \text{f}(x)$ and $y = \text{g}(x)$ are shown above.

(a) Find the value of

   (i) $\text{f}(-2)$,

Answer(a)(i) .... [1]

   (ii) $\text{g}(0)$.

Answer(a)(ii) .... [1]

(b) Use the graphs to solve

   (i) the equation f(x) = 20,

Answer(b)(i) $x =$ .... or $x =$ .... [2]

   (ii) the equation f(x) = g(x),

Answer(b)(ii) $x =$ .... or $x =$ .... [2]

   (iii) the inequality f(x) < g(x).

Answer(b)(iii) .... [1]

(c) Use the points *A* and *B* to find the gradient of $y =$ g(x) as an exact fraction.

Answer(c) .... [2]

(d) On the grid, draw the graph of $y =$ g(x) − 10. [2]

(e) **(i)** Draw the tangent to the graph of $y =$ f(x) at (-3, -27). [1]

   (ii) Write down the equation of this tangent.

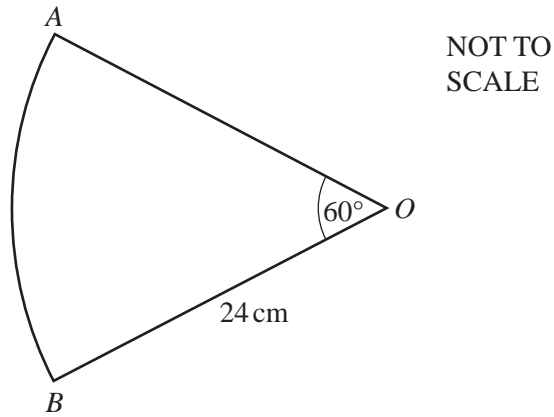*Answer(e)*(ii) .... [1]

(f) A region, *R*, contains points whose co-ordinates satisfy the inequalities

$$-3 \leqslant x \leqslant -2, \qquad y \leqslant 40 \qquad \text{and} \qquad y \geqslant g(x).$$

On the grid, draw suitable lines and label this region *R*. [2]

7

NOT TO
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60° *O*

24 cm

A

B

(a) The sector of a circle, centre *O*, radius 24 cm, has angle *AOB* = 60°.

Calculate

(i) the length of the arc *AB*,

Answer(a)(i) .... cm [2]

(ii) the area of the sector *OAB*.

Answer(a)(ii) .... cm^2 [2]

(b) The points *A* and *B* of the sector are joined together to make a hollow cone as shown in the diagram. The arc *AB* of the sector becomes the circumference of the base of the cone.

O

NOT TO
SCALE

24 cm

A

B

0580/04/O/N/09
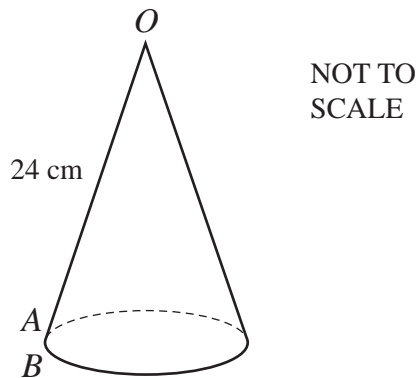
Calculate

(i) the radius of the base of the cone,

Answer(b)(i) .... cm [2]

(ii) the height of the cone,

Answer(b)(ii) .... cm [2]

(iii) the volume of the cone.

[The volume, *V*, of a cone of radius *r* and height *h* is $V = \frac{1}{3}\pi r^2 h$.]

Answer(b)(iii) .... cm^3 [2]

(c) A different cone, with radius *x* and height *y*, has a volume *W*.

Find, **in terms of *W***, the volume of

(i) a similar cone, with both radius and height 3 times larger,

Answer(c)(i) .... [1]

(ii) a cone of radius 2*x* and height *y*.

Answer(c)(ii) .... [1]

8 Fifty students are timed when running one kilometre.

The results are shown in the table.

Time (t minutes)	$4.0 < t \leqslant 4.5$	$4.5 < t \leqslant 5.0$	$5.0 < t \leqslant 5.5$	$5.5 < t \leqslant 6.0$	$6.0 < t \leqslant 6.5$	$6.5 < t \leqslant 7.0$
Frequency	2	7	8	18	10	5

(a) Write down the modal time interval.

Answer(a) .... min [1]

(b) Calculate an estimate of the mean time.

Answer(b) .... min [4]

(c) A new frequency table is made from the results shown in the table above.

Time (t minutes)	$4.0 < t \leqslant 5.5$	$5.5 < t \leqslant 6.0$	$6.0 < t \leqslant 7.0$
Frequency		18	

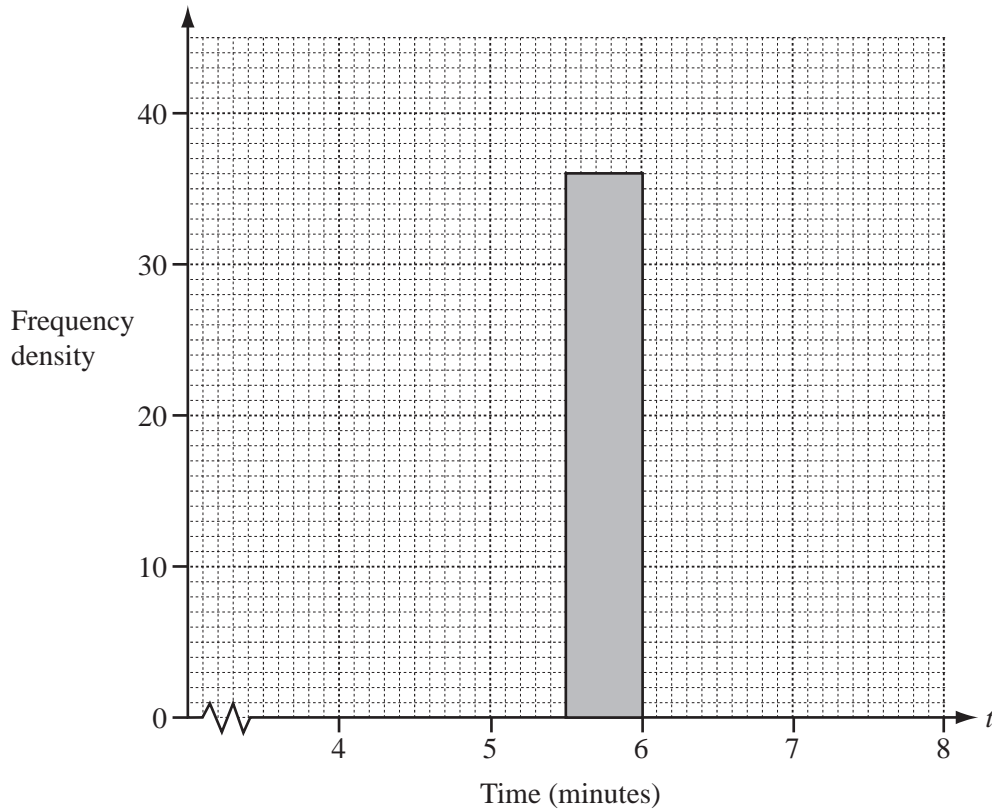**(i)** Complete the table by filling in the two empty boxes. [1]

(ii) On the grid below, complete an accurate histogram to show the information in this new table.

Frequency density (vertical axis): 0, 10, 20, 30, 40

Time (minutes) (horizontal axis, t): 4, 5, 6, 7, 8

[3]

(iii) Find the number of students represented by 1 cm² on the histogram.

Answer(c)(iii) .... [1]

[Turn over

9 **(a)** Solve the equation $\dfrac{m-3}{4} + \dfrac{m+4}{3} = -7$.

Answer(a) m = ... [4]

(b) **(i)** $y = \dfrac{3}{x-1} - \dfrac{2}{x+3}$

Find the value of y when $x = 5$.

Answer(b)(i) .... [1]

(ii) Write $\dfrac{3}{x-1} - \dfrac{2}{x+3}$ as a single fraction.

Answer(b)(ii) .... [2]

(iii) Solve the equation $\dfrac{3}{x-1} - \dfrac{2}{x+3} = \dfrac{1}{x}$.

Answer(b)(iii) $x =$ .... [3]

(c) $\qquad\qquad p = \dfrac{t}{q-1}$

Find q in terms of p and t.

Answer(c) q = [3]

10

Total

Row 1	1	=	1
Row 2	3 + 5	=	8
Row 3	7 + 9 + 11	=	27
Row 4	13 + 15 + 17 + 19	=	64
Row 5			
Row 6			

The rows above show sets of consecutive odd numbers and their totals.

(a) Complete Row 5 and Row 6. [2]

(b) What is the special name given to the numbers 1, 8, 27, 64…?

Answer(b) .... [1]

(c) Write down in terms of n,

   (i) how many consecutive odd numbers there are in Row n,

Answer(c)(i) .... [1]

   (ii) the total of these numbers.

Answer(c)(ii) .... [1]

(d) The first number in Row n is given by $n^2 - n + 1$.

Show that this formula is true for Row 4.

Answer(d)

[1]

(e) The total of Row 3 is 27. This can be calculated by $(3 \times 7) + 2 + 4$.

The total of Row 4 is 64. This can be calculated by $(4 \times 13) + 2 + 4 + 6$.

The total of Row 7 is 343. Show how this can be calculated in the same way.

Answer(e)

[1]

(f) The total of the first n **even** numbers is $n(n + 1)$.

Write down a formula for the total of the first $(n - 1)$ **even** numbers.

Answer(f) .... [1]

(g) Use the results of **parts (d), (e)** and **(f)** to show clearly that the total of the numbers in Row n gives your answer to **part (c)(ii)**.

Answer(g)

[2]

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