Cambridge
IGCSE

## Cambridge International Examinations

Cambridge International General Certificate of Secondary Education

## CANDIDATE

 NAMECENTRE NUMBER


CANDIDATE NUMBER

## CAMBRIDGE INTERNATIONAL MATHEMATICS

Paper 6 (Extended)

0607/63
May/June 2015
1 hour 30 minutes

Candidates answer on the Question Paper.
Additional Materials: Graphics Calculator

## READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO NOT WRITE IN ANY BARCODES.

Answer both parts A and B.
You must show all the relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 40 .

## Answer both parts A and B.

## A INVESTIGATION

## T-VALUES (20 marks)

You are advised to spend no more than 45 minutes on this part.
A grid of any length and width 10 is numbered $1,2,3, \ldots$.

|  | 2 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 10 | $5 n$ |

The grid has a letter T placed on it, as shown.
The T has a horizontal bar of length 3 and a vertical bar of length 2 .
The T shown is shape 1 because the number in the top left square of the T is 1 .
T -values are found using this method.

| Method | Calculation of T-value for shape 1 |
| :--- | :---: |
| Square the number at the bottom of the T. | $22^{2}=484$ |
| Multiply together the numbers at each end of the horizontal bar. | $1 \times 3=3$ |
| Take the second answer from the first answer to find the T-value. | $484-3=481$ |

The T -value for shape 1 is $\mathrm{T}_{1}=481$.
This investigation is about finding T -values.
1 (a) Complete this table.

| Shape number <br> $n$ | Working | T -value <br> $\mathrm{T}_{n}$ |
| :---: | :---: | :---: |
| 1 | $22^{2}-1 \times 3=484-3$ | $\mathrm{~T}_{1}=481$ |
| 2 | $23^{2}-2 \times 4=529-8$ | $\mathrm{~T}_{2}=521$ |
| 3 |  | $\mathrm{~T}_{3}=$ |
| 4 |  | $\mathrm{~T}_{4}=$ |
| 5 | $\mathrm{~T}_{5}=$ |  |

(b) When a T is placed at the end of a line, it still has a T -value. The T "wraps round" like this.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 42 | 44 | 45 | 46 | 47 | 18 | 10 | en |

Work out $\mathrm{T}_{9}$.

$$
\mathrm{T}_{9}=
$$

(c) $\mathrm{T}_{1}, \mathrm{~T}_{2}, \mathrm{~T}_{3}, \mathrm{~T}_{4}, \mathrm{~T}_{5}, \ldots$ form a sequence.

Find a formula, in terms of $n$, for $\mathrm{T}_{n}$.

$$
\mathrm{T}_{n}=
$$

(d) When $\mathrm{T}_{n}=2641$, find the value of $n$.

$$
n=
$$

$\qquad$
(e) Explain why 843 cannot be a T-value.
$\qquad$
$\qquad$

2 The T is now placed on a new grid that is 11 squares wide.

|  | 2 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
| 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 |
| 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 |
| 45 | 46 | 47 | 48 | 40 | 50 | 51 | 57 | 52 | 54 | 55 |

(a) Complete this statement for the numbers in the grid.

In each row the numbers increase by 1 and in each column the numbers increase by $\qquad$ . .
(b) Complete the squares in this T using expressions in terms of $n$.

(c) Complete this working to show that $\mathrm{T}_{n}=44 n+529$.

The first line of working is started for you.

$$
\mathrm{T}_{n}=(n+
$$

$\qquad$ . $)^{2}-n(n+$ $\qquad$

3 The T is now placed on a grid that is 12 squares wide.
Find a formula, in its simplest form, for $\mathrm{T}_{n}$.

$$
\mathrm{T}_{n}=
$$

4 The T is now placed on a grid that is $w$ squares wide.
(a) (i) Show that $\mathrm{T}_{n}=4 w^{2}+4(n+1) w+1$.
(ii) Find the width of the grid when $\mathrm{T}_{9}=1501$.
(b) Use part (a)(i) to explain why $\mathrm{T}_{n}$ must always be odd.

## B MODELLING

## FILLING A BATH (20 marks)

$$
\text { You are advised to spend no more than } 45 \text { minutes on this part. }
$$

Karen wants to work out how much it would cost to fill a bath every day for 1 year

1 She assumes that a bath is a cuboid that is 150 cm long, 60 cm wide and 30 cm deep.

(a) Find the number of litres of water needed to fill this bath to a depth of 20 cm .
$\qquad$ litres
(b) Water costs 20 cents per 100 litres.
(i) Find the cost, in dollars, of filling a bath with water to a depth of 20 cm each day for 1 year (365 days).
$\qquad$
(ii) Show that a model for the cost, $\$ C$, of filling a bath to a depth, $d \mathrm{~cm}$, every day for 1 year is

$$
C=6.57 d .
$$

(iii) In 1 year all baths are filled to the same depth, $d \mathrm{~cm}$. The total cost of the water is $\$ 157.68$.

Use the model in part (ii) to find the value of $d$.

You may use this information in the remainder of the task.

The area, $A$, of a trapezium is $\frac{h(a+b)}{2}$.


2 Karen notices that one end of her bath slopes.


From the side, her bath looks like this.


NOT TO
SCALE

She estimates that the angle of slope is $60^{\circ}$.
The depth of water in the bath is 20 cm .

150 cm


NOT TO
SCALE
(a) Show that $A B=5.77 \mathrm{~cm}$, correct to 3 significant figures.
(b) $B C=144 \mathrm{~cm}$, correct to 3 significant figures.

Find the volume of water in the bath, giving your answer in litres.
litres
(c) Show that a model for the volume of water in the bath, $V$ litres, with angle of slope $60^{\circ}$ and depth $d \mathrm{~cm}$ is

$$
V=0.03 d\left(300-\frac{(30-d)}{\tan 60^{\circ}}-\frac{30}{\tan 60^{\circ}}\right) .
$$

(d) On these axes, sketch the graph of $V$ for $0 \leqslant d \leqslant 30$.

(e) Find the depth of water in the bath when the volume is 150 litres.
$\qquad$

3 The depth of water in her bath is $d \mathrm{~cm}$.
Water costs $w$ cents per 100 litres.
(a) Change the model in question 2(c) to find the cost, $\$ C$, of filling her bath every day for 1 year.

$$
C=.
$$

(b) (i) Change your model in part (a) to find $C$ when the angle of slope is $\theta^{\circ}$.


$$
C=.
$$

(ii) Describe the effect on the volume of water in her bath when $\theta$ decreases.
$\qquad$
$\qquad$
(iii) Why is the design of this bath not suitable when $\theta$ is small?
$\qquad$
$\qquad$
(c) Use your model from part (b) to find the cost of filling her bath every day for 1 year when

- $\theta=50^{\circ}$
- $d=25 \mathrm{~cm}$
- water costs 21 cents per 100 litres.
$\qquad$

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