Kinematics (Chapter 1):

- Velocity: The rate of change of the displacement of an object.

$$\text{velocity} = \frac{\text{change in displacement}}{\text{time taken}}$$

An aircraft is flying due north with a velocity of 200 m s^{-1}. A side wind of velocity 50 m s^{-1} is blowing due east. What is the aircraft's resultant velocity (give the magnitude and direction)?

Here, the two velocities are at 90°. A sketch diagram and Pythagoras's theorem are enough to solve the problem.

Step 1 Draw a sketch of the situation – this is shown in Figure 1.16a.

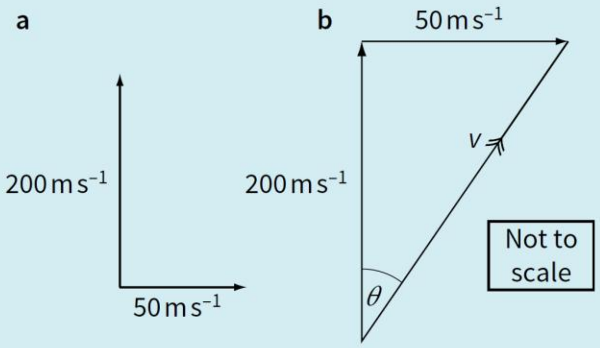


Figure 1.16 Finding the resultant of two velocities – for Worked example 5.

Step 2 Now sketch a vector triangle. Remember that the second vector starts where the first one ends. This is shown in Figure 1.16b.

Step 3 Join the start and end points to complete the triangle.

Step 4 Calculate the magnitude of the resultant vector v (the hypotenuse of the right-angled triangle).

$$v^2 = 200^2 + 50^2 = 40\,000 + 2500 = 42\,500$$

$$v = \sqrt{42\,500} \approx 206 \text{ m s}^{-1}$$

Step 5 Calculate the angle θ:

$$\tan\theta = \frac{50}{200}$$

$$= 0.25$$

$$\theta = \tan^{-1}(0.25) \approx 14°$$

So the aircraft's resultant velocity is 206 m s^{-1} at 14° east of north.

Accelerated Motion (Chapter 2):

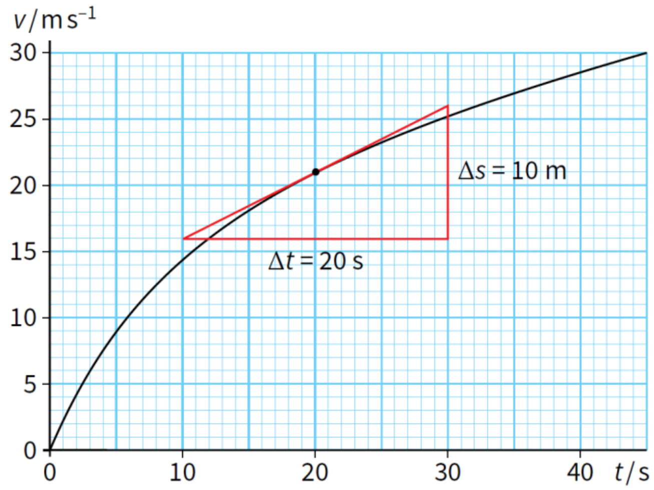
- Acceleration: rate of change of velocity

$$\text{average acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

- When an object is moving with constant acceleration in a straight line:

equation 1:	$v = u + at$
equation 2:	$s = \frac{(u+v)}{2} \times t$
equation 3:	$s = ut + \frac{1}{2}at^2$
equation 4:	$v^2 = u^2 + 2as$

- The velocity–time graph in Figure 2.18 shows non-uniform acceleration (decreasing gradient). The acceleration at any instant in time is given by the gradient of the velocity-time graph; calculated by:



- At the time of interest, mark a point on the graph.
- Draw a **tangent** to the curve at that point.
- Make a large right-angled triangle, and use it to find the gradient.

Figure 2.18 This curved velocity–time graph cannot be analysed using the equations of motion.

- In the absence of air resistance, the horizontal component of velocity is constant while the vertical component of velocity increases at a rate of -9.81 ms^{-2}.

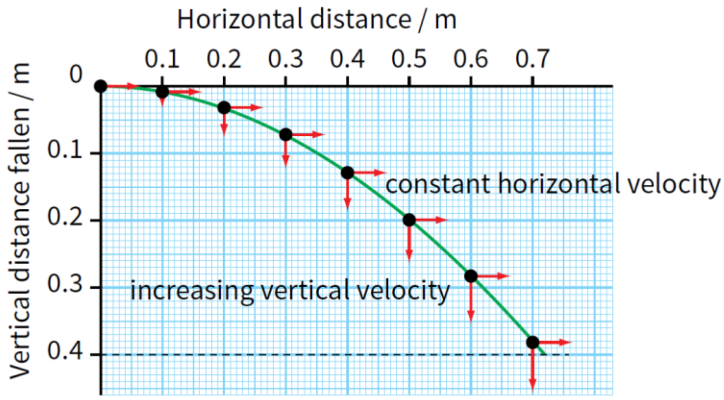


Figure 2.31 This sketch shows the path of the ball projected horizontally. The arrows represent the horizontal and vertical components of its velocity.

9 A stone is thrown horizontally with a velocity of 12 m s^{-1} from the top of a vertical cliff.

Calculate how long the stone takes to reach the ground 40 m below and how far the stone lands from the base of the cliff.

Step 1 Consider the ball's vertical motion. It has zero initial speed vertically and travels 40 m with acceleration 9.81 m s^{-2} in the same direction.

$s = ut + \frac{1}{2}at^2$

$40 = 0 + \frac{1}{2} \times 9.81 \times t^2$

Thus t = 2.86 s.

Step 2 Consider the ball's horizontal motion. The ball travels with a constant horizontal velocity, 12 m s^{-1}, as long as there is no air resistance.

distance travelled = $u \times t = 12 \times 2.86 = 34.3$ m

Hint: *You may find it easier to summarise the information like this:*

vertically	$s = 40$	$u = 0$	$a = 9.81$	$t = ?$	$v = ?$
horizontally	$u = 12$	$v = 12$	$a = 0$	$t = ?$	$s = ?$

10 A ball is thrown with an initial velocity of $20\,m\,s^{-1}$ at an angle of 30° to the horizontal (Figure 2.32). Calculate the horizontal distance travelled by the ball (its **range**).

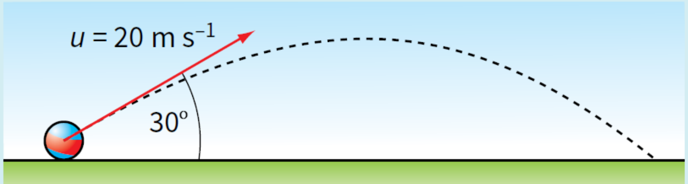


Figure 2.32 Where will the ball land?

Step 1 Split the ball's initial velocity into horizontal and vertical components:

initial velocity = $u = 20\,m\,s^{-1}$

horizontal component of initial velocity

$= u\cos\theta = 20 \times \cos 30° = 17.3\,m\,s^{-1}$

vertical component of initial velocity

$= u\sin\theta = 20 \times \sin 30° = 10\,m\,s^{-1}$

Step 2 Consider the ball's vertical motion. How long will it take to return to the ground? In other words, when will its displacement return to zero?

$u = 10\,m\,s^{-1}$ $\quad a = -9.81\,m\,s^{-2}$ $\quad s = 0$ $\quad t = ?$

Using $s = ut + \frac{1}{2}at^2$, we have:

$0 = 10t - 4.905t^2$

This gives $t = 0$ s or $t = 2.04$ s. So the ball is in the air for 2.04 s.

Step 3 Consider the ball's horizontal motion. How far will it travel horizontally in the 2.04 s before it lands? This is simple to calculate, since it moves with a constant horizontal velocity of $17.3\,m\,s^{-1}$.

horizontal displacement $s = 17.3 \times 2.04$

$= 35.3\,m$

Hence the horizontal distance travelled by the ball (its range) is about 35 m.

Dynamics (Chapter 3):

Base unit	Symbol	Base unit
length	x, l, s etc.	m (metre)
mass	m	kg (kilogram)
time	t	s (second)
electric current	I	A (ampere)
thermodynamic temperature	T	K (kelvin)
amount of substance	n	mol (mole)
luminous intensity	I	cd (candela)

- **Newton's second law of motion**: for a body of constant mass, its acceleration is directly proportional to the resultant force applied to it.

$F = ma$

- Difference between mass and weight:

Quantity	Symbol	Unit	Comment
mass	m	kg	this does not vary from place to place
weight	mg	N	this a force – it depends on the strength of gravity

- **Newton's first law of motion**: an object will remain at rest or in a state of uniform motion unless it is acted on by a resultant force.
- Terminal velocity: the maximum velocity of an object travelling through a fluid, where its resultant force is zero, and depends on the weight and surface area of the object.

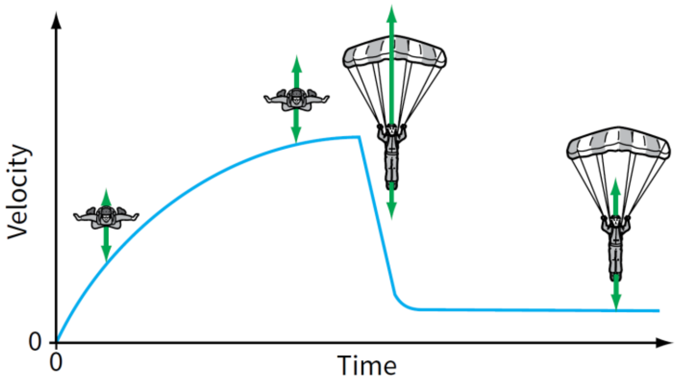


Figure 3.10 The velocity of a parachutist varies during a descent. The force arrows show weight (downwards) and air resistance (upwards).

- **Newton's third law of motion**: when two bodies interact, the forces they exert on each other are equal in magnitude and opposite in direction (action and reaction forces – of the same types).

Forces – vectors and moments (Chapter 4)

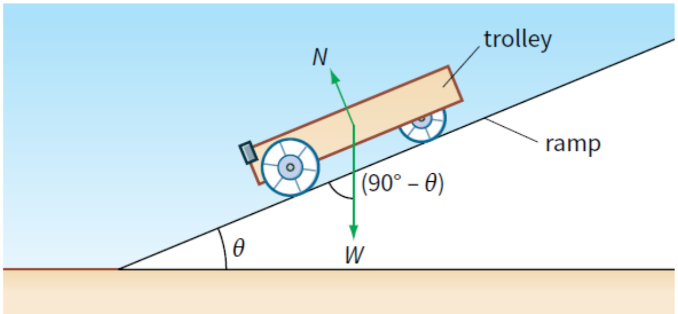


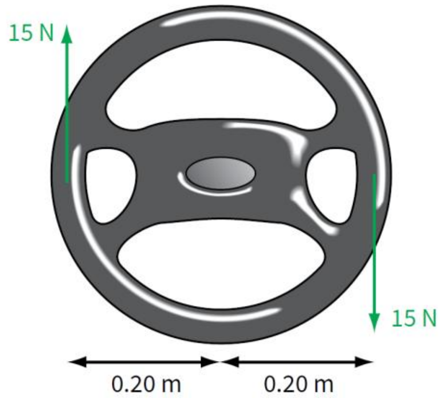
Figure 4.10 A force diagram for a trolley on a ramp.

$$\text{component of } W \text{ down the slope} = W\cos(90° - \theta) = W\sin\theta$$

$$F = mg\sin\theta$$

$$a = \frac{mg\sin\theta}{m} = g\sin\theta$$

- The centre of gravity of an object is defined as the point where all the weight of the object may be considered to act.
- The **moment** of a force = force × perpendicular distance of the pivot from the line of action of the force.
- **Principle of moments**: For any object that is in equilibrium, the sum of the clockwise moments about any point provided by the forces acting on the object equals to the sum of the anticlockwise moments about the same point.
- To form a couple, two forces must be equal in magnitude, parallel (but opposite in direction) and separated by a distance *d*
- The turning effect or moment of a couple is known as **torque**.
- **Torque of a couple** = one of the forces × perpendicular distance between the forces.



$$\text{torque of couple} = (15 \times 0.20) + (15 \times 0.20)$$
$$= 6.0\,\text{N m}$$

$$\text{torque of a couple} = 15 \times 0.4 = 6.0\,\text{N m}$$

- When calculating the moment, a pivot point must be chosen, where a force acts on.
- For an object to be in equilibrium, two conditions must be met at the same time:
  - The resultant force acting on the object is zero
  - The resultant moment is zero
- So a couple does not cause an object to accelerate.

Work, energy and power (Chapter 5)

- The **work done** by a force is defined as the product of the force and the distance moved in the direction of the force ($W = F \times s$).
- Work done = energy transferred

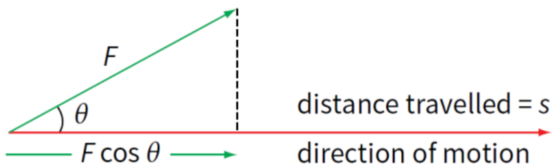


Figure 5.6 The work done by a force depends on the angle between the force and the distance it moves.

- Work done by an expanding gas: $W = p\Delta V$
- Gravitational potential energy (E_p) = mgh
  - h is the distance moved
  - mg is the force (weight) on the object
- Kinetic energy (E_k) = ½ mv^2
  - $v^2 = u^2 + 2as$ ($u = 0$)
  - $v^2 = 2as$
  - ½ $mv^2 = mas$
  - Work done by force F = ½ mv^2

$$\text{efficiency} = \frac{\text{useful output energy}}{\text{total input energy}} \times 100\%$$

- Principle of conservation of energy: energy cannot be created or destroyed. It can only be converted from one form to another.
- **Power** is defined as the rate of work done, units: watt

$$\text{power} = \frac{\text{work done}}{\text{time taken}} \qquad P = \frac{W}{t}$$

- Suppose that an aircraft is moving with velocity *v*. Its engines provide the force *F* needed to overcome the drag of the air. In time *t*, the aircraft moves a distance *s* equal to *vt*. So the work done by the engines is:

$$\text{work done} = \text{force} \times \text{distance}$$

$$W = F \times v \times t$$

and the power P $\left(= \frac{\text{work done}}{\text{time taken}}\right)$ is given by:

$$P = \frac{W}{t} = \frac{F \times v \times t}{t}$$

and we have:

$$P = F \times v$$

$$\text{power} = \text{force} \times \text{velocity}$$

Momentum (Chapter 6)

- Momentum = mass × velocity ($p = mv$)
- The principle of conservation of momentum:
  - Within a closed system, the total momentum in any direction is constant.
  - Total momentum of objects before collision = total momentum of objects after collision.
- Perfectly elastic collision (does not stick):
  - Momentum conserved
    - m1u1 + m2u2 = m1v1 + m2v2
  - E_k conserved

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \text{—} ① \quad \text{(conservation of momentum)}$$

$$\Rightarrow m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad \text{—} ②$$

$$\cancel{\tfrac{1}{2}}m_1u_1^2 + \cancel{\tfrac{1}{2}}m_2u_2^2 = \cancel{\tfrac{1}{2}}m_1v_1^2 + \cancel{\tfrac{1}{2}}m_2v_2^2 \quad \text{—} ③ \quad \text{(Elastic collision)}$$

$$\Rightarrow m_1u_1^2 + m_2u_2^2 = m_1v_1^2 + m_2v_2^2$$

$$\Rightarrow m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

$$m_1(u_1 + v_1)(u_1 - v_1) = m_2(v_2 + u_2)(v_2 - u_2) \quad \text{—} ④$$

$$④ \div ② \Rightarrow u_1 + v_1 = v_2 + u_2$$

$$\therefore u_1 - u_2 = v_2 - v_1$$

$$\boxed{u_1 - u_2 = -(v_1 - v_2)}$$

- Perfectly inelastic collision (does stick):
  - Momentum conserved
    - m1u1 + m2u2 = (m1 + m2)(v1 + v2)

- E_k not conserved

3 A white ball of mass $m = 1.0$ kg and moving with initial speed $u = 0.5\,\mathrm{m\,s^{-1}}$ collides with a stationary red ball of the same mass. They move off so that each has the same speed and the angle between their paths is 90°. What is their speed?

Step 1 Draw a diagram to show the velocity vectors of the two balls, before and after the collision (Figure 6.16). We will show the white ball initially travelling along the *y*-direction.

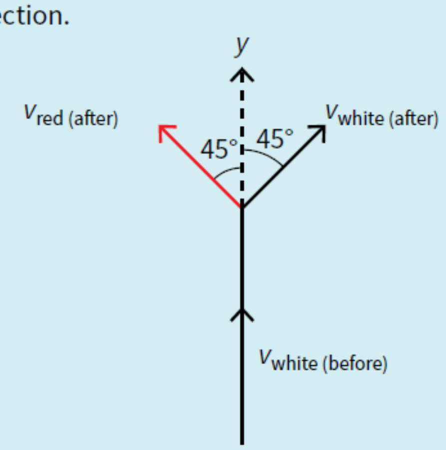


Figure 6.16 Velocity vectors for the white and red balls.

Because we know that the two balls have the same final speed *v*, their paths must be symmetrical about the *y*-direction. Since their paths are at 90° to one other, each must be at 45° to the *y*-direction.

Step 2 We know that momentum is conserved in the *y*-direction. Hence we can say:

initial momentum of white ball in *y*-direction
= final component of momentum of white ball in *y*-direction
+ final component of momentum of red ball in *y*-direction

This is easier to understand using symbols:

$mu = mv_y + mv_y$

where v_y is the component of *v* in the *y*-direction. The right-hand side of this equation has two identical terms, one for the white ball and one for the red. We can simplify the equation to give:

$mu = 2mv_y$

Step 3 The component of *v* in the *y*-direction is $v\cos 45°$. Substituting this, and including values of *m* and *u*, gives

$0.5 = 2v\cos 45°$

and hence

$$v = \frac{0.5}{2\cos 45°} \approx 0.354\,\mathrm{m\,s^{-1}}$$

So each ball moves off at $0.354\,\mathrm{m\,s^{-1}}$ at an angle of 45° to the initial direction of the white ball.

4 Figure 6.17 shows the momentum vectors for particles 1 and 2, before and after a collision. Show that momentum is conserved in this collision.

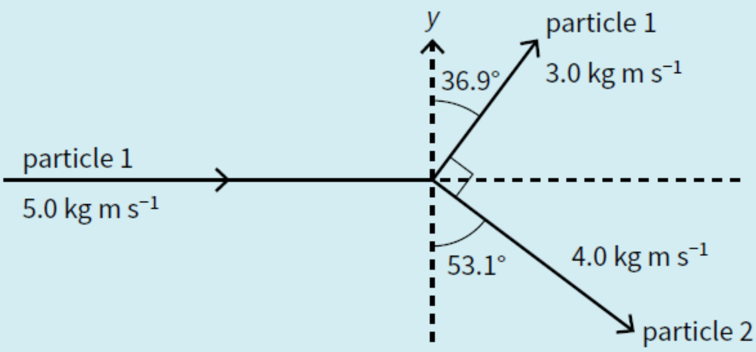


Figure 6.17 Momentum vectors: particle 1 has come from the left and collided with particle 2.

Step 1 Consider momentum changes in the *y*-direction.

Before collision:

momentum = 0

(because particle 1 is moving in the *x*-direction and particle 2 is stationary).

After collision:

component of momentum of particle 1
$= 3.0\cos 36.9° \approx 2.40\,\mathrm{kg\,m\,s^{-1}}$ upwards

component of momentum of particle 2
$= 4.0\cos 53.1° \approx 2.40\,\mathrm{kg\,m\,s^{-1}}$ downwards

These components are equal and opposite and hence their sum is zero. Hence momentum is conserved in the *y*-direction.

Step 2 Consider momentum changes in the *x*-direction.

Before collision: momentum $= 5.0\,\mathrm{kg\,m\,s^{-1}}$ to the right

After collision:

component of momentum of particle 1
$= 3.0\cos 53.1° \approx 1.80\,\mathrm{kg\,m\,s^{-1}}$ to the right

component of momentum of particle 2
$= 4.0\cos 36.9° \approx 3.20\,\mathrm{kg\,m\,s^{-1}}$ to the right

total momentum to the right $= 5.0\,\mathrm{kg\,m\,s^{-1}}$

Hence momentum is conserved in the *x*-direction.

Step 3 An alternative approach would be to draw a vector triangle similar to Figure 6.15b. In this case, the numbers have been chosen to make this easy; the vectors form a 3–4–5 right-angled triangle.

Because the vectors form a closed triangle, we can conclude that:

momentum before collision = momentum after collision
i.e. momentum is conserved.

- Newton's second law of motion: the resultant force acting on an object is equal to the rate of change of its linear momentum (force = rate of change of momentum). The resultant force and the change in momentum are in the same direction.

$$F = \frac{\Delta p}{\Delta t}$$

$F = ma$ used when the mass of the object stays constant.

Matter and materials (Chapter 7)

- Density is defined as the mass per unit volume of a substance

$$\rho = \frac{m}{v}$$

- Pressure is defined as the normal force acting per unit cross-sectional area

$$p = \frac{F}{A}$$

- Pressure in a fluid is given by:

$$\text{weight of water} = \text{mass} \times g = \rho \times A \times h \times g$$

$$\text{pressure} = \frac{\text{force}}{\text{area}} = \rho \times A \times h \times \frac{g}{A}$$

$$= \rho \times g \times h$$

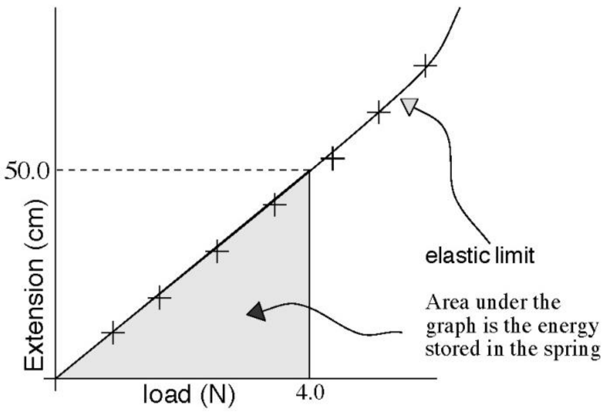
$$\text{pressure} = \text{density} \times \text{acceleration due to gravity} \times \text{depth}$$

$$p = \rho g h$$

- Hooke's law: a material obeys Hooke's law if the extension produced in it is proportional to the applied force (load).

$F = kx$

  - Where k is the force constant of the spring / stiffness / spring constant
- The force beyond which the spring becomes permanently deformed is known as the **elastic limit**.



- The **strain** is defined as the fractional increase in the original length of the wire; has no units; hence the usage of a long wire – increase extension

$$\text{strain} = \frac{\text{extension}}{\text{original length}} \qquad \text{strain} = \frac{x}{L}$$

- The **stress** is defined as the force applied per unit cross-sectional area of the wire; units: Pa; hence the usage of a thin wire – increase extension

$$\text{stress} = \frac{\text{force}}{\text{cross-sectional area}} \qquad \text{stress} = \frac{F}{A}$$

- **Young modulus**: ratio of stress to strain of a material

$$E = \frac{\text{tensile stress}}{\text{tensile strain}} \qquad \text{Young modulus} = \frac{\text{stress}}{\text{strain}}$$

$$= \frac{\sigma}{\varepsilon}$$

$$= \frac{F/A}{e/L}$$

$$= \frac{FL}{eA}$$

- If the linear section stress is proportional to strain, the wire obeys Hooke's law:

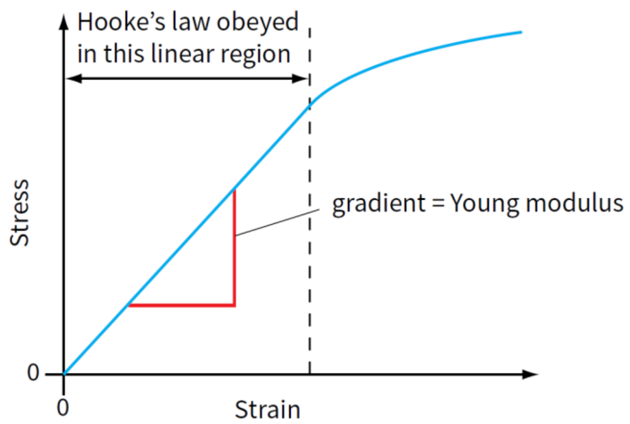


Figure 7.11 Stress–strain graph, and how to deduce the Young modulus. Note that we can only use the first, straight-line section of the graph.

- **Elastic potential energy**: energy stored in a stretched or compressed material.
    - ➢ As long as the elastic limit has not been exceeded, the energy can be recovered.
- The work done in stretching or compressing a material is equal to the area under force-extension graph.

$$\text{area} = \tfrac{1}{2} \times \text{base} \times \text{height}$$

This again gives:

$$\text{elastic potential energy} = \tfrac{1}{2}Fx$$

or $$E = \tfrac{1}{2}Fx$$

There is an alternative equation for elastic potential energy. We know that, according to Hooke's law (page 104), applied force F and extension x are related by $F = kx$, where k is the force constant. Substituting for F gives:

$$\text{elastic potential energy} = \tfrac{1}{2}Fx = \tfrac{1}{2} \times kx \times x$$

$$\text{elastic potential energy} = \tfrac{1}{2}kx^2$$

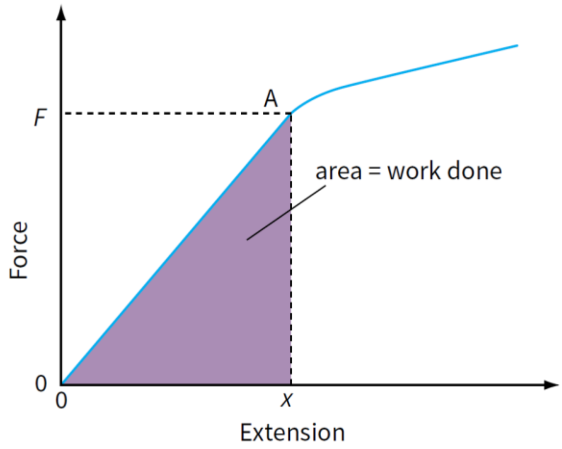


Figure 7.15 Elastic potential energy is equal to the area under the force–extension graph.