Cambridge International Examinations
Cambridge Ordinary Level

ADDITIONAL MATHEMATICS **4037/22**

Paper 2 **October/November 2016**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

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Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied
www without wrong working

Question	Answer	Marks	Part Marks
1	$4x-3=x \rightarrow x=1$	**B1**	www
	$4x-3=-x$	**M1**	use of $-x$ or $-(4x-3)$ but not both.
	$x=0.6$	**A1**	
	OR $(4x-3)^2=x^2$	**B1**	
	$15x^2-24x+9=0$		
	$3(x-1)(5x-3)=0$	**M1**	solve correct 3 term quadratic
	$x=1$ and $x=0.6$	**A1**	www
2	$a(\sqrt{3}-1)+b(\sqrt{3}+1)$ $=(\sqrt{3}-3)(\sqrt{3}-1)(\sqrt{3}+1)$ $=2(\sqrt{3}-3)$ oe	**M1**	Common denominator or $\times(\sqrt{3}-1)(\sqrt{3}+1)$
	$a+b=2$	**DM1**	equate constant terms and $\sqrt{3}$ terms.
	$-a+b=-6$	**A1**	both correct
		DM1	solve two **linear** equations to obtain $a=$ or $b=$
	$b=-2$ and $a=4$	**A1**	both correct
3	$2\lg x=\lg x^2$	**B1**	soi anywhere
	$1=\lg 10$	**B1**	soi anywhere
	$\lg x^2-\lg\left(\frac{x+10}{2}\right)=\lg\left(\frac{2x^2}{x+10}\right)$ oe	**B1**	soi division; logs may be removed
	$2x^2-10x-100=0 \rightarrow 2(x+5)(x-10)=0$	**M1**	obtain correct 3 term quadratic equation and attempt to solve
	$x=10$ only	**A1**	$x=-5$ must not remain.

Question	Answer	Marks	Part Marks
4 (i)	$t=10 \rightarrow N=7000+2000\mathrm{e}^{-0.5}$ $=8213$ or 8210	B1	Do not accept non integer responses.
(ii)	$N=7500 \rightarrow 7500=7000+2000\mathrm{e}^{-0.05t}$ $\mathrm{e}^{-0.05t}=\frac{500}{2000}$ $-0.05t=\ln 0.25 \rightarrow t=\frac{\ln 0.25}{-0.05}$ $=27.7$ (days)	M1 M1 A1	insert and make $\mathrm{e}^{-0.05t}$ subject take logs and make t the subject awrt 27.7
(iii)	$\frac{\mathrm{d}N}{\mathrm{d}t}=-100\mathrm{e}^{-0.05t}$ $t=8 \rightarrow \frac{\mathrm{d}N}{\mathrm{d}t}=\pm 67$ (.0)	M1 A1 A1	$k\mathrm{e}^{-0.05t}$ where k is a constant $k=-100$ or -0.05×2000 awrt ± 67 mark final answer
5 (i)	$\frac{\mathrm{d}y}{\mathrm{d}x}=3x^2+4x-7$ $x=-2 \rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=12-8-7=-3$ Equation of tangent : $\frac{y-16}{x+2}=-3 \rightarrow y=-3x+10$	B1 M1 A1	 insert $x=-2$ into *their* gradient and use $(-2, 16)$ and *their* gradient of tangent in equation of line.
(ii)	Tangent cuts curve again $x^3+2x^2-7x+2=-3x+10$ $x^3+2x^2-4x-8=0$ $(x+2)(x+2)(x-2)=0$ $x=2, \quad y=4$	M1 A1 M1 A1A1	equate curve and *their* linear answer from (i). factorise: $(x\pm 2)$ and a two or three term quadratic is sufficient. Allow long division withhold final **A1** if (2, 4) not clearly identified as their sole answer.
6 (i)	$\frac{\cos x}{1+\tan x}-\frac{\sin x}{1+\cot x}=\frac{\cos x}{1+\frac{\sin x}{\cos x}}-\frac{\sin x}{1+\frac{\cos x}{\sin x}}$ $=\frac{\cos^2 x}{\cos x+\sin x}-\frac{\sin^2 x}{\cos x+\sin x}$ $=\frac{(\cos x-\sin x)(\cos x+\sin x)}{(\cos x+\sin x)}$	M1 M1 A1 A1	$\tan x=\frac{\sin x}{\cos x}$ and $\cot x=\frac{\cos x}{\sin x}$ Attempt to multiply by $\cos x$ and $\sin x$ AG
(ii)	$-\sin x+\cos x=3\sin x-4\cos x$ $5\cos x=4\sin x$ $\tan x=\frac{5}{4}$ $x=51.3°, -128.7°$	M1 A1 A1A1	equate and collect $\sin x$ and $\cos x$ oe **FT** from tan $x=k$

Question	Answer	Marks	Part Marks
7 (i)	$h=\sqrt{9-x^2}$ $A=\frac{\sqrt{9-x^2}}{2}(14+x+x)=\sqrt{9-x^2}\,(7+x)$	B2/1/0	Must be clear that $\sqrt{9-x^2}$ is the height of the trapezium. $14+2x$ oe must be seen AG
(ii)	$\frac{\mathrm{d}A}{\mathrm{d}x}=\sqrt{9-x^2}+(7+x)\frac{1}{2}\left(9-x^2\right)^{-0.5}\times -2x$	M1 A2/1/0	product rule on correct function minus 1 each error , allow unsimplified.
	$\frac{\mathrm{d}A}{\mathrm{d}x}=0\rightarrow 9-x^2=7x+x^2$ $2x^2+7x-9=0$	M1 A1	equate to 0 and simplify to a linear or quadratic equation. correct three term quadratic obtained
	$x=1$ $A=16\sqrt{2}$ or $8\sqrt{8}$ or $\sqrt{512}$ or 22.6	A1 A1	Extra positive answer loses penultimate **A1**. ignore negative solution.
8 (i)	$\mathrm{f}'(x)=\frac{\left(x^3+1\right)9x^2-\left(3x^3-1\right)3x^2}{\left(x^3+1\right)^2}$	M1 A1	quotient rule or product rule all correct
	$=\frac{12x^2}{\left(x^3+1\right)^2}$	A1	www beware $9x^6-9x^6$ gets **A0**
(ii)	$\int_1^2\frac{x^2}{\left(x^3+1\right)^2}\mathrm{d}x=\frac{1}{12}\left[\frac{3x^3-1}{x^3+1}\right]_1^2$	M1 A1	$c\times\frac{3x^3-1}{x^3+1}$ **FT** $c=\frac{1}{\mathit{their}\,12}$
	$=\frac{1}{12}\left[\frac{23}{9}-\frac{2}{2}\right]$	DM1	top limit – bottom limit in *their* integral.
	$=\frac{7}{54}$	A1	or 0.130 or 0.1296 or 0.12
(iii)	$x=\frac{3y^3-1}{y^3+1}$ $y^3=\frac{x+1}{3-x}$	B1	make y^3 or x^3 the subject
	$\mathrm{f}^{-1}(x)=\sqrt[3]{\frac{x+1}{3-x}}$	B1 B1	**FT** take cube root (as long as y^3 or x^3 equals a fraction with terms in x or y only) oe **FT** change x and y – can be done at any time
	Domain : $-1\leqslant x\leqslant 2\frac{6}{7}$	B1	Allow upper limit of 2.86 . Do not isw

Question	Answer	Marks	Part Marks
9 (i)	tangent touches circle $x^2+(kx-4)^2-2(kx-4)=8$	M1	eliminate y or x allow unsimplified
	$k^2x^2+x^2-8kx-2kx+16=0$ or better	A1	
	Equal roots as tangent touches circle : $b^2=4ac$	DM1	use of discriminant on 3 term quadratic soi
	$(-10k)^2=4(k^2+1)\times 16$	A1	
	$36k^2=64$		
	$k=+\frac{4}{3}$ only	A1	oe any inequality loses last **A1**
(ii)	$x=\frac{-b}{2a}$ $\rightarrow x=\frac{\frac{4}{3}\times 10}{\frac{25}{9}}$	M1	use $x=\frac{-b}{2a}$
	$x=\frac{12}{5}$ $y=-\frac{4}{5}$	A1A1	
	OR tangent $y=\frac{4}{3}x-4$ cuts radius	M1	find equation of radius and attempt to solve with tangent
	$y=-\frac{3}{4}x+1$		
	at $x=\frac{12}{5}$	A1	
	$y=-\frac{4}{5}$	A1	
	OR Obtain $25x^2-120x+144=0$ oe	M1	obtain any 3 term quadratic using *their* non zero *k* and reach $x=\ldots$
	$(5x-12)(5x-12)=0$		
	$x=\frac{12}{5}\rightarrow y=-\frac{4}{5}$	A1A1	
(iii)	$TP=\sqrt{(0-2.4)^2+(-4+0.8)^2}=4$	M1A1	**M1** for using *their T* and $(0,-4)$. Signs must be correct.

Question	Answer	Marks	Part Marks
10 (i)	$r_j = \begin{pmatrix} 5000 \\ 1000p \end{pmatrix} + \begin{pmatrix} -2\cos 40 \\ 2\cos 50 \end{pmatrix} t$	**B1** **B1**	x coordinate oe y coordinate oe
(ii)	$2.5t\cos 70 = 5000 - 2t\cos 40$	**M1**	equate *their* x values (must be 3 terms)
	$t = \dfrac{5000}{2.5\cos 70 + 2\cos 40}$	**DM1**	make t the subject allow one sign error
	$= 2095$ awrt or 2090 or 2100	**A1**	
	$(2.5\cos 20 - 2\cos 50) \times 2095 = 1000p$	**M1**	equate *their* y values(must be 3 terms) and insert *their* t or $\lvert t \rvert$.
	$p = 2.23$ awrt	**A1**	
11 (i)	Free choice : no. of ways ${}^6C_4 \times {}^5C_2 = 15 \times 10$ $= 150$	**B1** **B1**	${}^6C_4 \times$ another ${}^nC_r$ term only $\times {}^5C_2$ and answer or vice versa
(ii)	Both Mr and Mrs Coldicott ${}^5C_3 \times {}^4C_1 = 10 \times 4$ $= 40$	**B1** **B1**	${}^5C_3 \times$ another ${}^nC_r$ term only $\times {}^4C_1$ and answer or vice versa
(iii)	Mr C and not Mrs C ${}^5C_3 \times {}^4C_2\ (= 60)$ Not Mr C and Mrs C ${}^5C_4 \times {}^4C_1\ (= 20)$ Total = 80	**B1** **B1** **B1**	An incorrect final answer does not affect the awarding of the first two **B1** marks. www
	OR Total = (i) – (ii) – neither Neither $= {}^5C_4 \times {}^4C_2 = 30$ Total = 150 – 40 – 30 = 80	**M1** **A1** **A1**	