## Cambridge International Examinations

Cambridge Pre-U Certificate

## MATHEMATICS (PRINCIPAL)

Paper 1 Pure Mathematics 1

## Additional Materials: Answer Booklet/Paper

Graph Paper
List of Formulae (MF20)

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet. Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [] at the end of each question or part question.
The total number of marks for this paper is 80 .

1 Find the set of values of $x$ for which $x^{2}-x-12<0$.

2 Solve the following simultaneous equations.

$$
\begin{equation*}
x^{2}+2 y^{2}=36 \quad x+2 y=10 \tag{5}
\end{equation*}
$$

3


The diagram shows a triangle $A B C$ in which angle $B=39^{\circ}$, angle $C=28^{\circ}, A B=x \mathrm{~cm}$ and $A C=(2 x-1) \mathrm{cm}$. Find the value of $x$.

4 A population, $P$, is modelled by the equation $P=a \mathrm{e}^{b t}$ where $t$ is time in years, and $a$ and $b$ are constants.
(i) By considering logarithms, show that a graph of $\ln P$ against $t$ is a straight line. State the intercept on the vertical axis and the gradient.
(ii) Use the graph below to obtain values for $a$ and $b$.


5 A circle has equation $x^{2}-6 x+y^{2}-4 y=12$.
(i) Show that the centre of the circle is at the point $(3,2)$ and find the radius.
(ii) $P Q$ is a diameter of the circle where $P$ has coordinates $(-1,-1)$. Find the equation of $P Q$, giving your answer in the form $a x+b y=c$ where $a, b$ and $c$ are integers.
(iii) Another diameter of the circle passes through the point $(0,6)$. Show that this diameter is perpendicular to $P Q$.

6 The functions f and g are given by $\mathrm{f}(x)=\frac{3}{x-1}$ for all $x \neq 1$ and $\mathrm{g}(x)=x+2$ for all real $x$.
(i) Find gf, stating its domain and range.
(ii) Find $(\mathrm{gf})^{-1}$, stating any values of $x$ for which $(\mathrm{gf})^{-1}$ is not defined.

7 The lines $l_{1}$ and $l_{2}$ have the following vector equations.

$$
\begin{aligned}
& l_{1}: \mathbf{r}=3 \mathbf{i}+2 \mathbf{j}+\mathbf{k}+\lambda(\mathbf{i}-6 \mathbf{j}-2 \mathbf{k}) \\
& l_{2}: \mathbf{r}=\mathbf{i}+5 \mathbf{j}+2 \mathbf{k}+\mu(3 \mathbf{j}+\mathbf{k})
\end{aligned}
$$

(i) Show that the lines $l_{1}$ and $l_{2}$ intersect and find the coordinates of their point of intersection.
(ii) Find the acute angle between the lines $l_{1}$ and $l_{2}$.

8 The complex numbers $w$ and $z$ are given by $w=3-\mathrm{i}$ and $z=1+\mathrm{i}$.
(i) Express $\frac{z}{w}$ in the form $p+\mathrm{i} q$ where $p$ and $q$ are real numbers.
(ii) On the same Argand diagram, mark the points representing $z, w$ and $\frac{z}{w}$.
(iii) Find the value in radians of $\arg w$.
(iv) Show that $z+\frac{2}{z}$ is a real number.

9 A curve has equation $y=\left(x^{2}-3\right) \mathrm{e}^{-x}$. Find the exact coordinates of the stationary points of the curve.

10 A curve has parametric equations given by

$$
x=-\sqrt{(1-t)^{3}} \quad y=\sqrt{(1+t)^{3}} \quad \text { for }-1<t<1
$$

(i) Show that $\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{1+t}{\sqrt{1-t^{2}}}$.
(ii) Write $\frac{\mathrm{d} y}{\mathrm{~d} x}$ as a series of ascending powers of $t$ up to and including the term in $t^{4}$, and hence estimate the gradient of the curve when $t=0.5$.

11 Using the substitution $x=u^{2}-1$, or otherwise, show that

$$
\begin{equation*}
\int \frac{1}{2 x \sqrt{x+1}} \mathrm{~d} x=\ln \left(A \sqrt{\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}}\right) \tag{10}
\end{equation*}
$$

where $A$ is an arbitrary constant and $x>0$.

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