## Cambridge International Examinations

Cambridge Pre-U Certificate

## MATHEMATICS (PRINCIPAL)

Paper 2 Pure Mathematics 2

## Additional Materials: Answer Booklet/Paper

 Graph PaperList of Formulae (MF20)

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet. Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80 .

1 Find the equation of the line which passes through the points $(2,5)$ and $(8,-1)$. Show that this line also passes through the point $(-2,9)$.
(a) (i) Find the value of the discriminant of $x^{2}+3 x+5$.
(ii) Use your value from part (i) to determine the number of real roots of the equation $x^{2}+3 x+5=0$.
(b) Find the non-zero value of $k$ for which the equation $k x^{2}+3 x+5=0$ has only one distinct real root.

3 Solve the equation $\tan \left(\theta+10^{\circ}\right)=0.1$ in the range $0^{\circ} \leqslant \theta \leqslant 360^{\circ}$.

4 A sequence of complex numbers is defined by

$$
u_{1}=1+\mathrm{i} \quad \text { and } \quad u_{n+1}=\mathrm{i} u_{n} \quad(n=1,2,3, \ldots)
$$

(i) Find $u_{2}, u_{3}, u_{4}, u_{5}$ and $u_{6}$.
(ii) Describe the behaviour of the sequence.
(iii) Hence evaluate $\sum_{n=1}^{73} u_{n}$.

5 (i) Differentiate $\frac{x}{\sqrt{1+x^{2}}}$ with respect to $x$.
(ii) Hence show that $\frac{x}{\sqrt{1+x^{2}}}$ is increasing for all $x$.

6 Find the solution of the differential equation

$$
\begin{equation*}
x y^{2} \frac{\mathrm{~d} y}{\mathrm{~d} x}=x+1 \tag{7}
\end{equation*}
$$

given that $y=3$ when $x=1$. Give your answer in the form $y=\mathrm{f}(x)$.

7 A curve, $C$, is given parametrically by $x=2 \cos \theta, y=3 \sin \theta, 0<\theta<\frac{1}{2} \pi$.
(i) Show that $\frac{\mathrm{d} y}{\mathrm{~d} x}=-\frac{3}{2} \cot \theta$.

A tangent to $C$ intersects the $x$-axis and $y$-axis at $P$ and $Q$ respectively.
(ii) Show that the midpoint of $P Q$ has coordinates $\left(\sec \theta, \frac{3}{2} \operatorname{cosec} \theta\right)$.
(iii) Hence show that the midpoint of $P Q$ lies on the curve $\frac{4}{x^{2}}+\frac{9}{y^{2}}=4$.

8 (i) Express $\frac{7 x^{2}-12 x+1}{\left(x^{2}+1\right)(x-2)}$ in the form $\frac{A x+B}{x^{2}+1}+\frac{C}{x-2}$ where $A, B$ and $C$ are constants to be found.
(ii) Hence find the exact value of $\int_{0}^{1} \frac{7 x^{2}-12 x+1}{\left(x^{2}+1\right)(x-2)} \mathrm{d} x$.

9 (i) Show that $\int x(x-2)^{\frac{3}{2}} \mathrm{~d} x=\frac{2}{35}(5 x+4)(x-2)^{\frac{5}{2}}+c$.
(ii) Hence find the coordinates of the stationary points of the curve

$$
\begin{equation*}
y=\frac{2}{35}(5 x+4)(x-2)^{\frac{5}{2}}+x^{2}-\frac{1}{3} x^{3} . \tag{6}
\end{equation*}
$$

10 An arithmetic sequence and a geometric sequence have $n$th terms $a_{n}$ and $g_{n}$ respectively, where $n=1,2,3, \ldots$. It is given that $a_{1}=g_{1}, a_{2}=g_{2}, a_{5}=g_{3}, a_{1} \neq a_{2}$ and $a_{1} \neq 0$.
(i) Show that the common ratio of the geometric sequence is 3 .
(ii) Find the common difference of the arithmetic sequence in terms of $a_{1}$.
(iii) Let $a_{1}=g_{1}=5$.
(a) Find the first three terms of both sequences.
(b) Show that every term of the geometric sequence is also a term of the arithmetic sequence.

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